Extended Failed-Literal Preprocessing for Quantified Boolean Formulas

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Software directory, contains QxbfCntrsSat11.tar.gz
as well as individual files.

The Problem:

In a Quantified Boolean Formula (QBF), find literals that must have a certain truth value and pairs of literals that must have the same truth value.

Main Ideas in This Paper:

- Derive pairs of binary clauses that contain only two variables among them.
- Use the pure universal literal rule, but not the pure existential literal rule.

Related Work:

J. W. Freeman, "Failed literals in the Davis-Putnam procedure for SAT", *DIMACS Challenge Workshop* 1993.

D. Le Berre, "Exploiting the real power of unit propagation", *LICS Sat Workshop*, 2001.

F. Lonsing and A. Biere, "Failed literal detection for QBF", *SAT* 2011.

What are Quantified Boolean Formulas (QBFs)?

Most general definition:

- Add quantification ($\forall u$, $\exists e$) as a new propositional operation.
- A quantified variable must be *true* or *false*.

Least general definition:

- $\mathcal{F}$ is a quantifier-free propositional formula in *conjunctive normal form*.
- $\overrightarrow{Q}$ is a sequence of quantified variables, outer to inner scopes.
- $\Phi = \overrightarrow{Q}\,.\,\mathcal{F}$ is a ***prefix CNF*** (PCNF) formula.
- This paper considers *closed* PCNF (all variables quantified).
- Example (chart form):

Φ	$\forall u$	$\forall v$	$\exists e$	$\exists f$	$\forall t$	$\exists d$
C_1	u			f	t	$\overline{d}$
C_2	$\overline{u}$	$\overline{v}$	$\overline{e}$		$\overline{t}$	$\overline{d}$
C_3		v	e	f	t	d
$\dots$						

Literal Naming Convention

- Lowercase letters near the beginning of the alphabet are *existential* literals (or variables, if specified in the context), e.g., c, d, e, etc.
- Lowercase letters near the end of the alphabet are *universal* literals (or variables, if specified in the context), e.g., t, u, v, etc.
- p, q, r, s are of unspecified quantifier type.
- $|p|$ denotes the *variable* underlying the *literal* p.
  (Mainly for quantifier sequences.)

QBF as a Two-Player Game

Two players: A is *Universal*, E is *Existential*.

$\Phi = \overrightarrow{Q}\,.\,\mathcal{F}$ is a PCNF (prefix: $\overrightarrow{Q}$, matrix: $\mathcal{F}$).

When outermost (unassigned) variable is *universal*, A chooses a value for it and Φ gets simplified.

When outermost (unassigned) variable is *existential*, E chooses a value for it and Φ gets simplified.

If Φ simplifies to *false*, A wins. If Φ simplifies to *true*, E wins.

"Definition": *Truth-Value Semantics* of Φ (coarse grain):

- The value of Φ is *false* (or 0) if and only if A has a winning strategy.
- The value of Φ is *true* (or 1) if and only if E has a winning strategy.

Example: Two-Player Game

Φ	qblock $\leftarrow 1 \rightarrow$		qblock $\leftarrow 2 \rightarrow$		qblock 3	qblock 4
	$\forall u$	$\forall v$	$\exists e$	$\exists f$	$\forall t$	$\exists d$
C_1	u			f	t	$\overline{d}$
C_2	$\overline{u}$	$\overline{v}$	$\overline{e}$		$\overline{t}$	$\overline{d}$
C_3		v	e	f	t	d
$\ldots$						

Say A chooses $u = 0$;
C_2 is "satisfied" and
is deleted from $\mathcal{F}$.
u is deleted from C_1.

Then say A chooses $v = 0$;
v is deleted from C_3.

Φ_1	$\exists e$	$\exists f$	$\forall t$	$\exists d$
C_1		f	t	$\overline{d}$
C_3	e	f	t	d
$\ldots$				

Next, Player E chooses values
for e and f.

And so on.

Notes:

- Quantifier order matters between qblocks, but not within a qblock.
- *One* empty clause makes Φ false (the goal of A).
- *Every* clause must be satisfied to make Φ true (the goal of E), but *one* true literal suffices to satisfy a clause.

Tree Models for QBF

For a QBF $\Phi = \overrightarrow{Q} \cdot \mathcal{F}$, a ***tree model*** of Φ is a nonempty set of ***ordered assignments*** with certain restrictions that ensure that the set defines a tree.

An ***ordered assignment*** is a total assignment with the literals in quantifier-prefix order, outer to inner.

Each ordered assignment is a tree branch and satisfies all clauses.

Φ	$\forall u$	$\forall v$	$\exists e$	$\exists f$	$\forall t$	$\exists d$
C_1	u			f	t	$\overline{d}$
C_2	$\overline{u}$	$\overline{v}$	$\overline{e}$		$\overline{t}$	$\overline{d}$
C_3		v	e	f	t	d
	$\ldots$					
	$\overline{u}$	$\overline{v}$	e	f	$\overline{t}$	$\overline{d}$
	$\overline{u}$	$\overline{v}$	e	f	t	d
	$\overline{u}$	v	e	f	$\overline{t}$	$\overline{d}$
	$\overline{u}$	v	e	f	t	d
	u	$\overline{v}$	e	f	$\overline{t}$	$\overline{d}$
	u	$\overline{v}$	e	f	t	d
	u	v	$\overline{e}$	f	$\overline{t}$	$\overline{d}$
	u	v	$\overline{e}$	f	t	d

The 8 ordered assignments shown are a tree model for the part of Φ shown. There are many others.

Q-Resolution

Two operations:

- *propositional resolution* on an *existential* clashing literal (tautologous resolvents prohibited);
  - Let $C_1 = [e, \alpha]$ and $C_2 = [\overline{e}, \beta]$.
  - $\mathbf{res}_e(C_1, C_2) = \alpha \cup \beta$.

- *universal reduction* on a *tailing universal* literal.
  - If u is inner scope to all existentials in clause C, it may be deleted (locally assigned *false* in C only).
  - Let $C = [u, \gamma]$.
  - $\mathbf{unrd}_u(C) = \gamma$.

Theorem [Kleine Büning,Karpinski,Flögel 1995]:

A PCNF Φ is *false* if and only if the empty clause is derivable by Q-resolution.

Remark (speaker's opinion):

Universal reduction is a major factor in recent success of practical QBF solvers.

Failed Literals and 1-Saturation: **Propositional Calculus**

Failed Literal Rule

If assuming p leads to a contradiction by unit propagation, infer $\overline{p}$.

Double look-ahead:

If assuming p leads to unit-clause q and assuming $\overline{p}$ leads to unit-clause q,
then infer q.

If assuming p leads to unit-clause q and assuming $\overline{p}$ leads to unit-clause $\overline{q}$,
then infer $p = q$.

0-saturation (J. Freeman, later G. Stålmarck):

Do double look-ahead for each variable.

1-saturation (G. Stålmarck, later Le Berre):

Do double look-ahead for each variable.

Failed Literals and 1-Saturation: QBF

Lonsing and Biere, SAT 2011:

- The Failed Literal Rule is *unsound* on QBF if p is not outermost scope.
- *Abstraction* of *Phi* w.r.t p consists of making all variables outer to p existential.
- Only *0-saturation* was conducted in `QxBF`.

This paper:

- Combination of pure existential literal rule and pure universal literal rule and abstraction is unsound for double look-ahead (example in proceedings).

- Combination of *pure universal literal rule* and abstraction *is* sound for double look-ahead (proved in proceedings).

Interactions and Effects on Tree Models

Abstraction makes Φ "truer" (more tree models).

Pure existential literal rule makes Φ "falser" (fewer tree models).

Pure universal literal rule makes Φ "falser" (fewer tree models).

Unit-clause propagation and universal reduction do not change the set of tree models.

So ? ? ?

If abstraction changes u from universal to existential, and then it becomes pure, making it *true* is the wrong idea.

Say u is assigned *true* due to purity.
u cannot imply anything by unit propagation.
But u can make some *universal* literal, say v, become pure.
This combination is unsound.

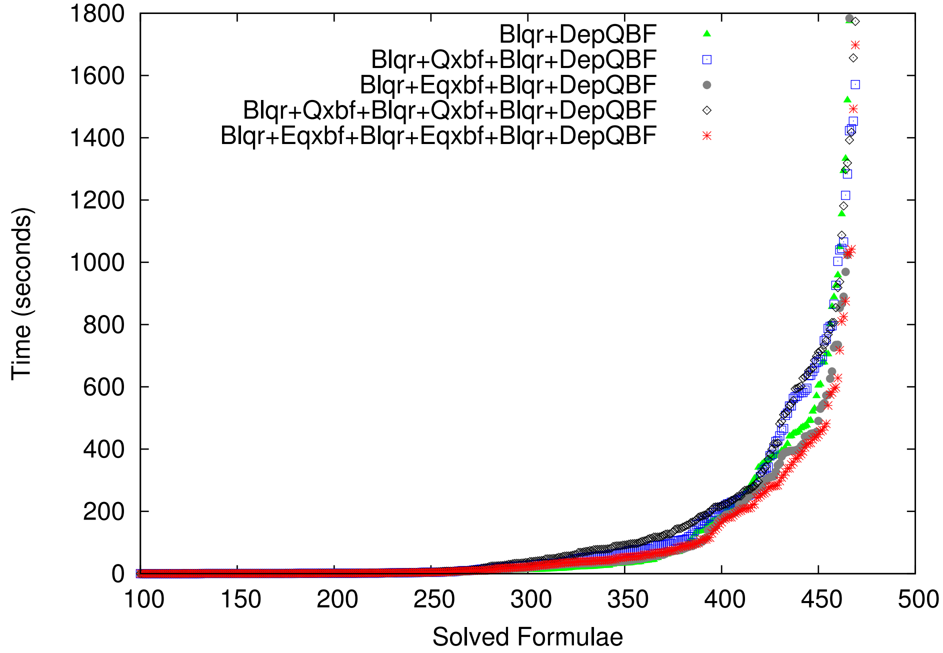
Experimental Results (more details in proceedings)

Extended QxBF adds 1-saturation with pure universal literal rule to QxBF.

Combine exqbf and bloqqer (Biere, Seidl, Lonsing, CADE 2011).

Run multiple rounds so bloqqer can digest exqbf results.

depqbf is the back-end solver.



Conclusion

eqxbf produced modest improvements.

More blueQBFEVAL-10 instances solved by preprocessing alone (191 vs. 148).

Combination solver in QBFEVAL-12 as hiqqer.

Better understanding of QBF theory.

Role and value of pure existential literal rule should be re-examined.