Off the trail: re-examining the CDCL algorithm.

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Overview

- Using the standard (historical) description, the trail seems vital to the completeness of DPLL.
- Paradigms and descriptions change
- The trail is not as important as it may seem
  - Is simply an efficient way of deciding which variable to flip
  - No inherent importance to the algorithm
- Current CDCL algorithms can be reformulated as local search with a complicated heuristic

DPLL Algorithm

Given: ϕ - a formula in CNF

Solve(ϕ)

$\phi \leftarrow$ unitPropagate(ϕ)

if ϕ reduces to a constant, return ϕ

v $\leftarrow$ 'best' unassigned var

return Solve($\phi|_{v}$) $\vee$ Solve($\phi|_{\neg v}$)

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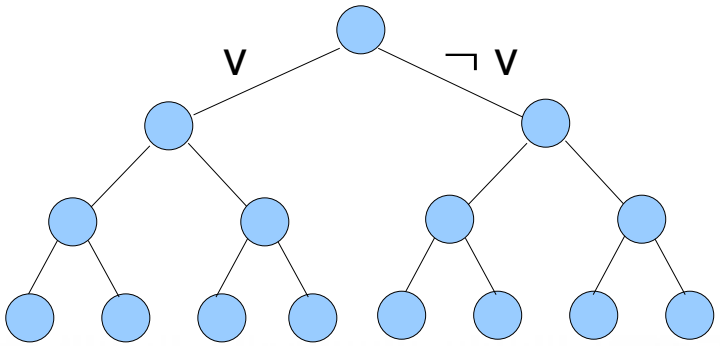

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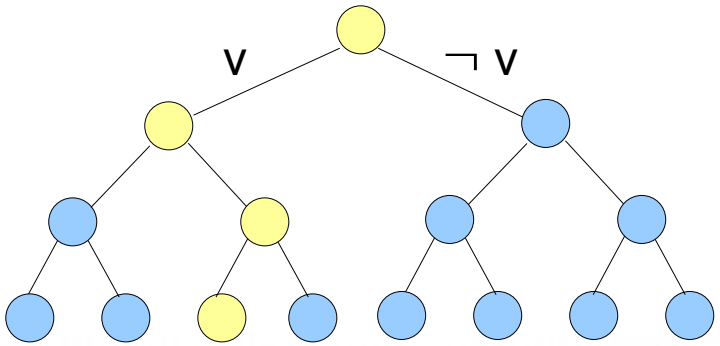

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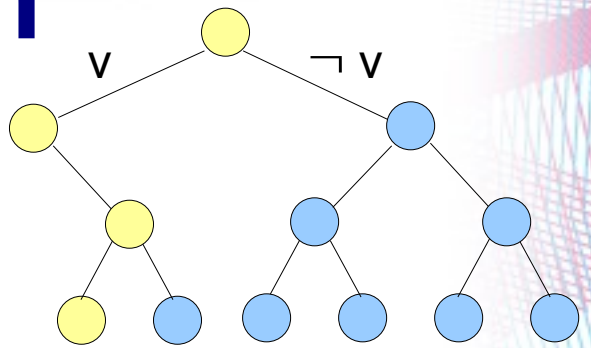

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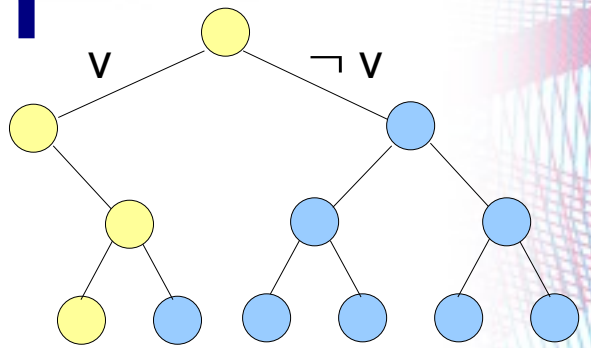

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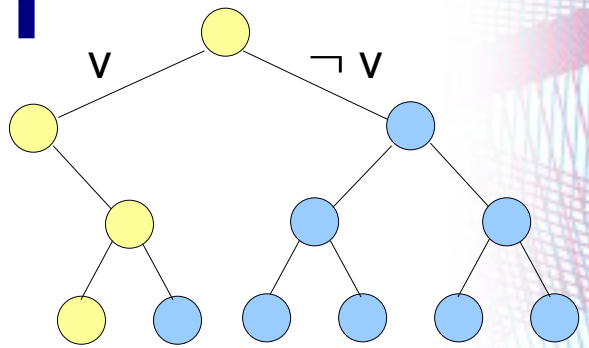

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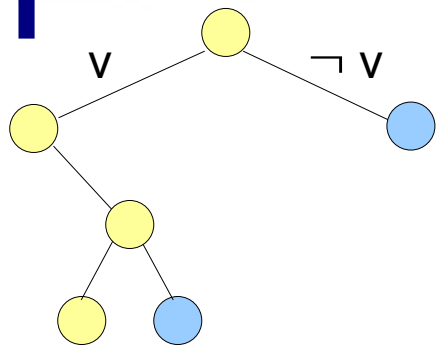

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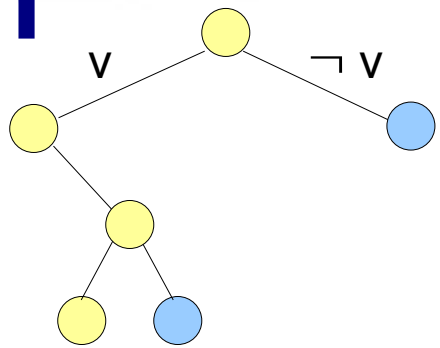

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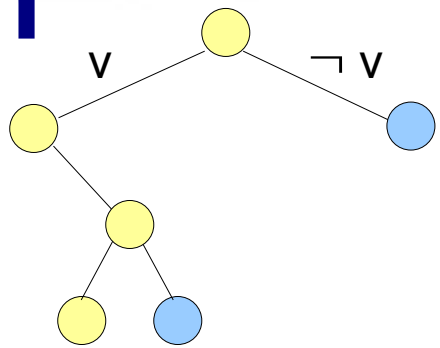
Clause learning:

- **Learn from conflicts**
- ***Empowering* clauses**
- **Without deletion, guarantees completeness**

DPLL Algorithm



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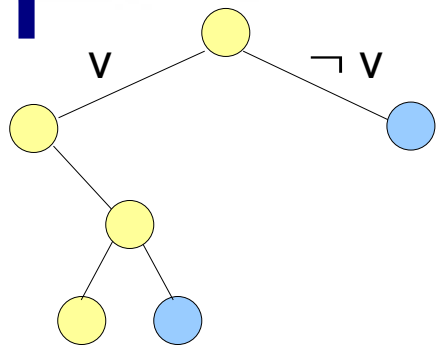

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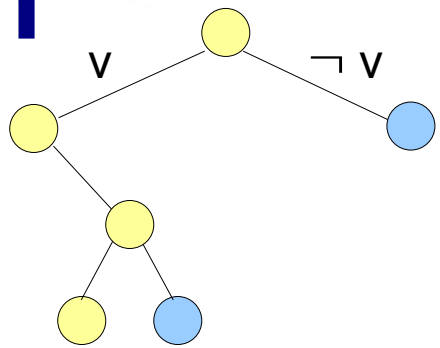

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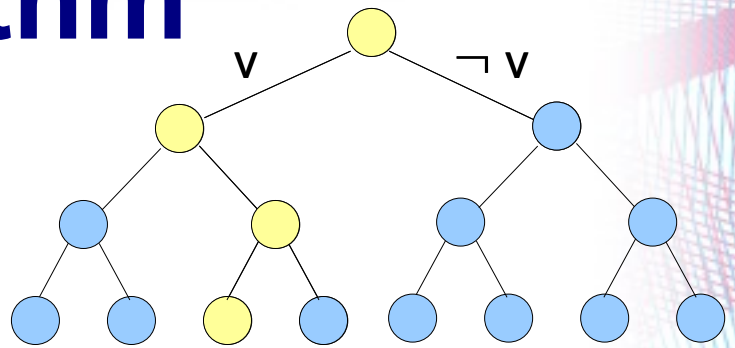

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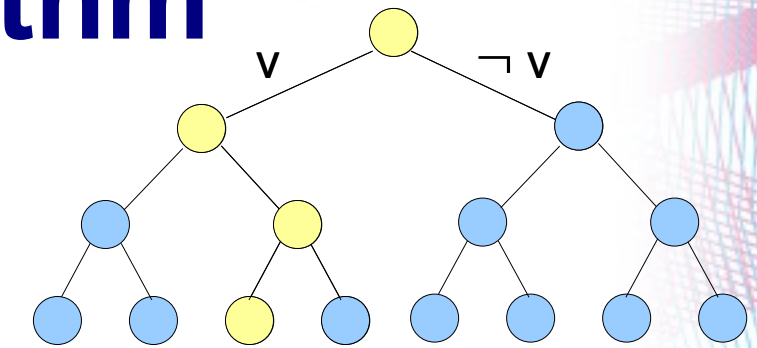
DPLL Algorithm

Restart to a different part of search space

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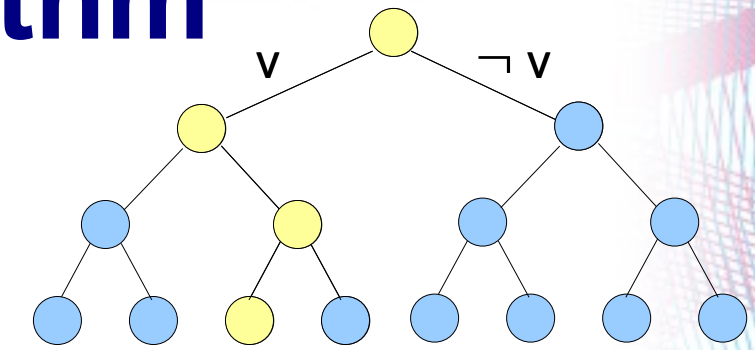
Restart to a different part of search space
+ Progress saving

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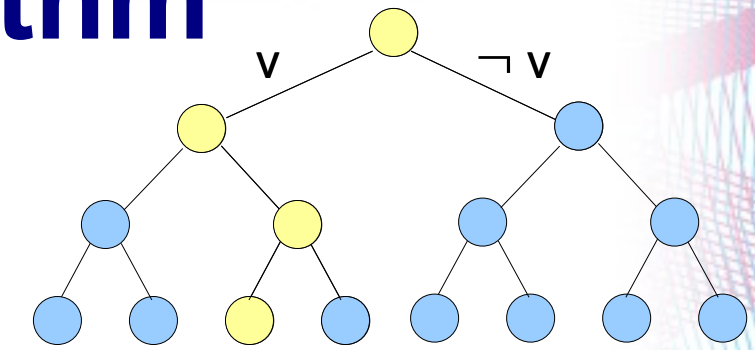


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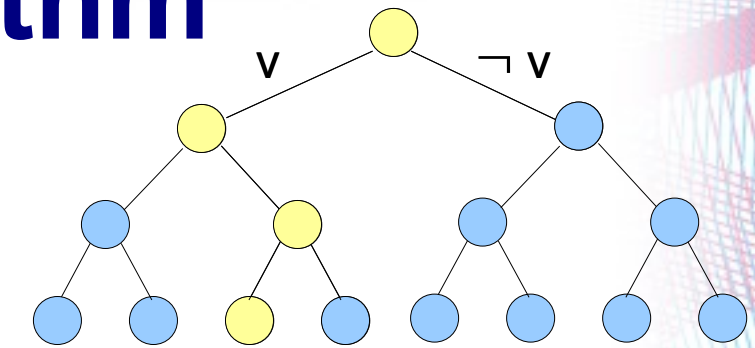

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  c(2.5)
d(4.1)
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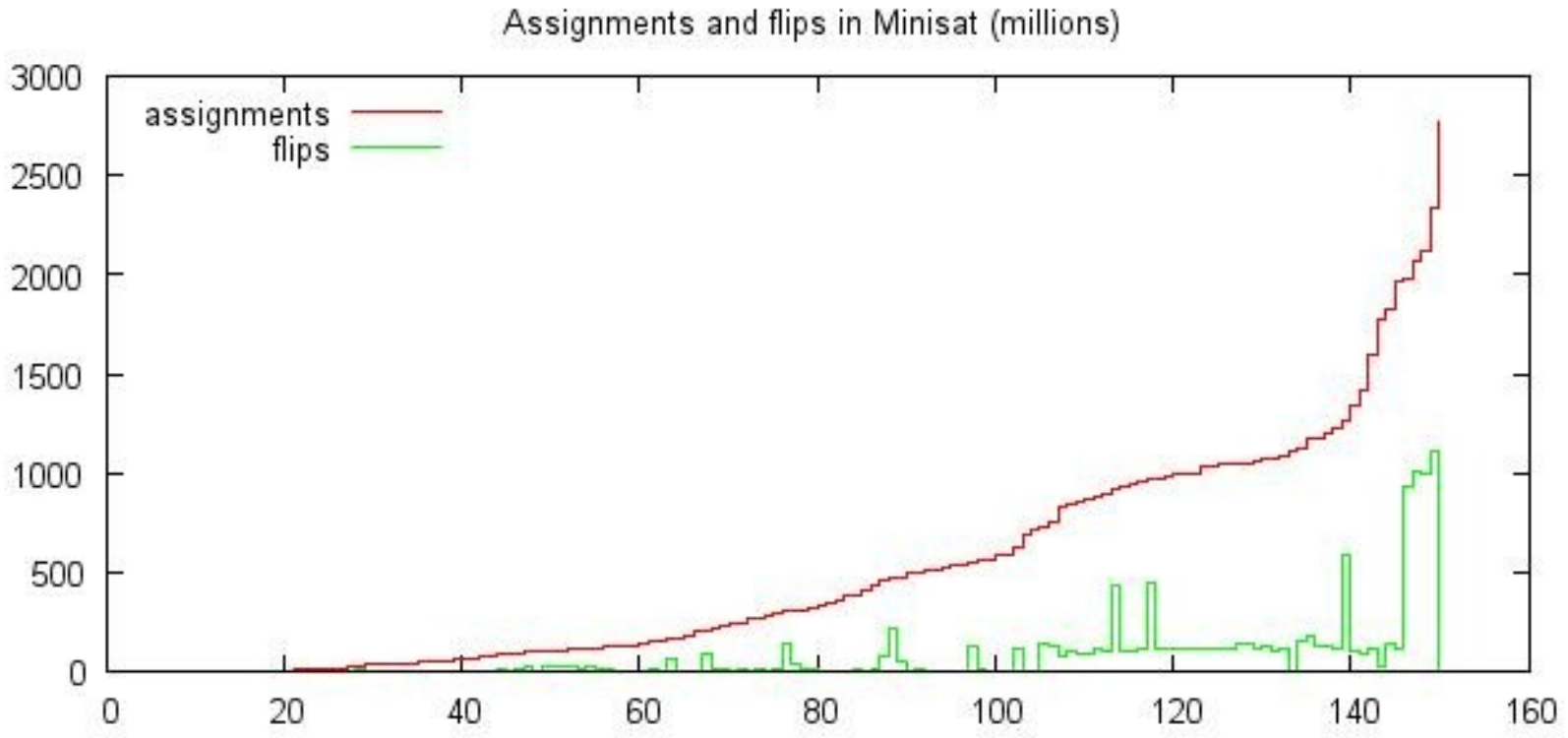
Effect of restart: the trail is updated to order decisions by heuristic

Assignments vs. flips



- **Assignment: any time a variable gets a value**
- **Flip: any time a variable gets a value that is *different from its phase setting***

Concentrate on Flips

- Imagine a complete assignment:
  - Variable's value, if it exists
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  - Every flip has a *reason:*
    - A clause which contains the flipped variable
    - The flipped variable is the “least important” in that clause.

Intuition

- Variables with high heuristic values are “important”
- A false clause means a flip. We want to flip the least important variable.
- However: if ($\alpha \rightarrow$ x) and we flip x, then:
  - Either we'll need to flip x back
  - Or we'll need to flip a variable from α
- So: any variable implied by important variables is important

Local Search

Given: ϕ - a formula in CNF

```
while(φ|_I is not TRUE)

    I← initValues()
    while φ|_I contains FALSE clauses
        if timeToRestart() then continue outer
        v ← pickVar(I)
        flip(v)

return TRUE
```

Local Search

- Want to get “closer to a solution”
  - estimating is very difficult
  - estimates are unreliable
  - lots of local minima
- Escaping local minima:
  - generate a new clause that would change the landscape
  - if a clause is guaranteed to be new, get completeness

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- For better clauses, want an implication graph.
- How to generate it from existing assignment I?
  - CDLS: “simulate UP until it first deviates from I”

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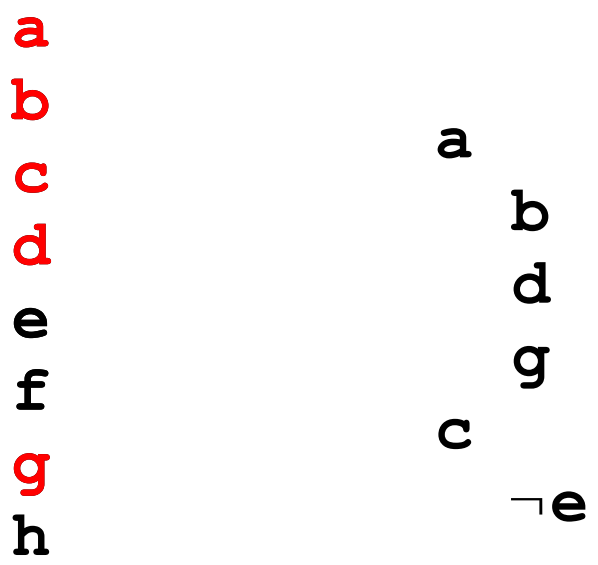

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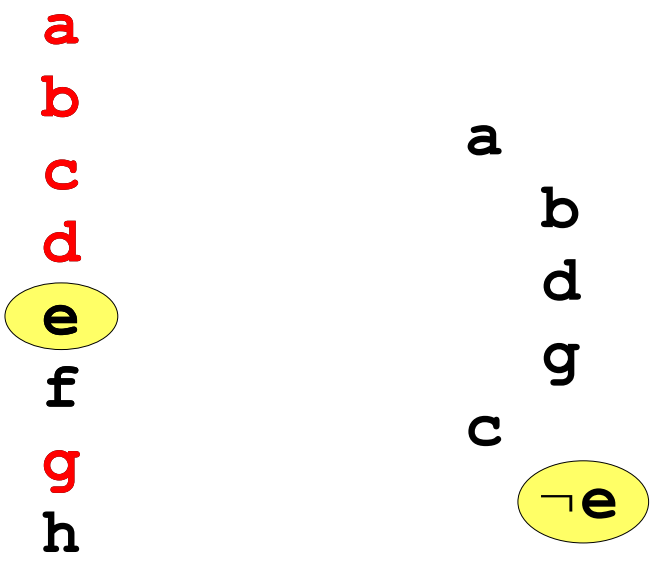

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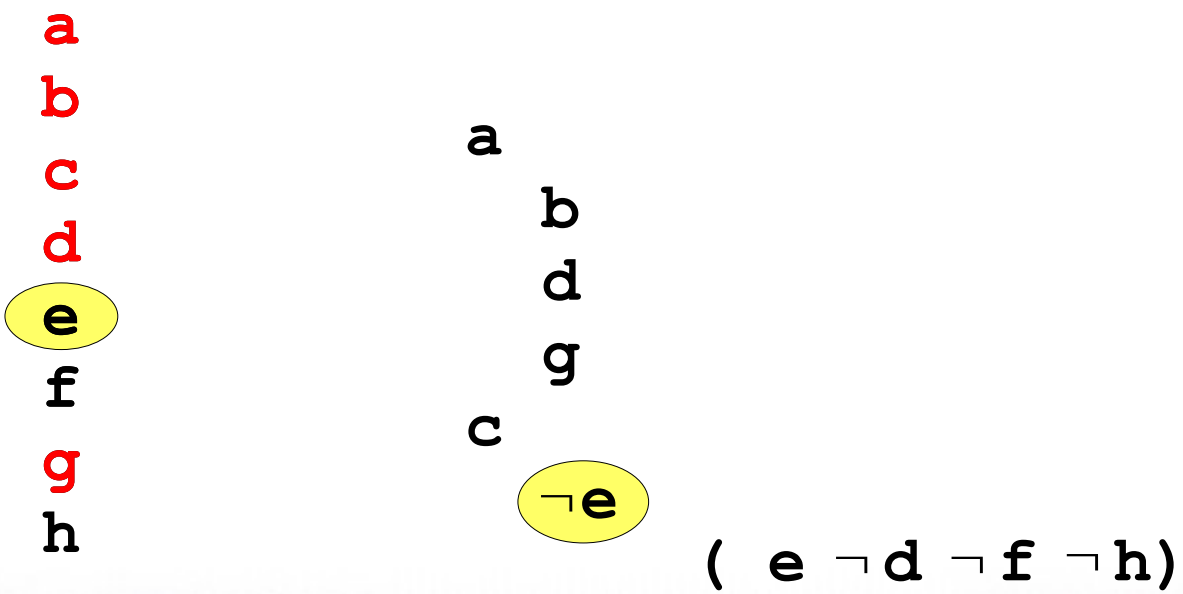

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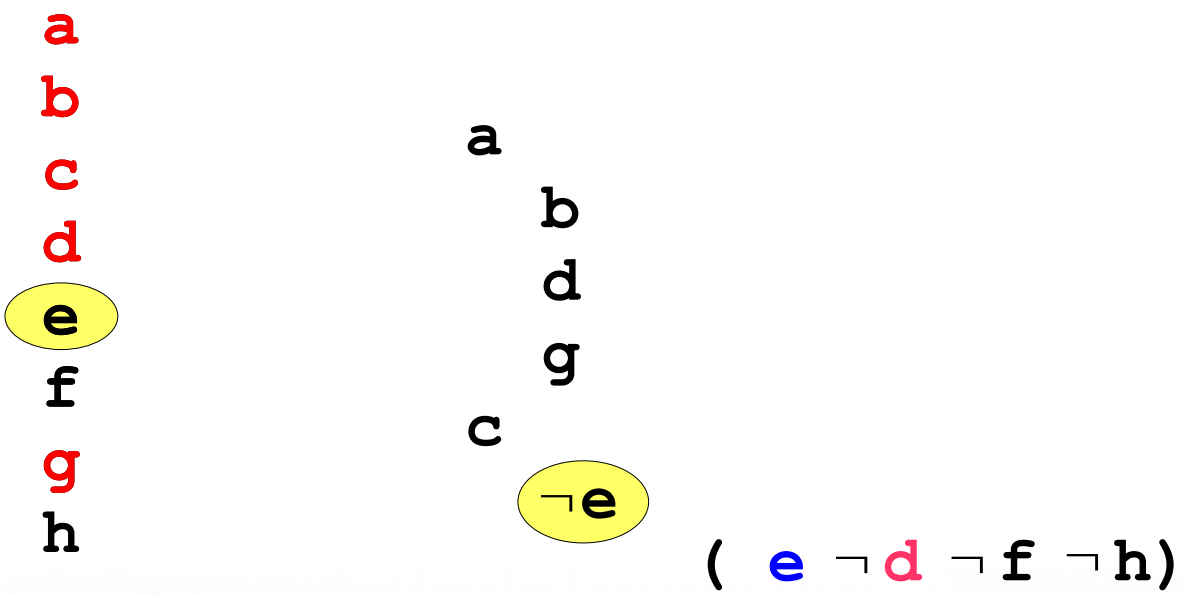

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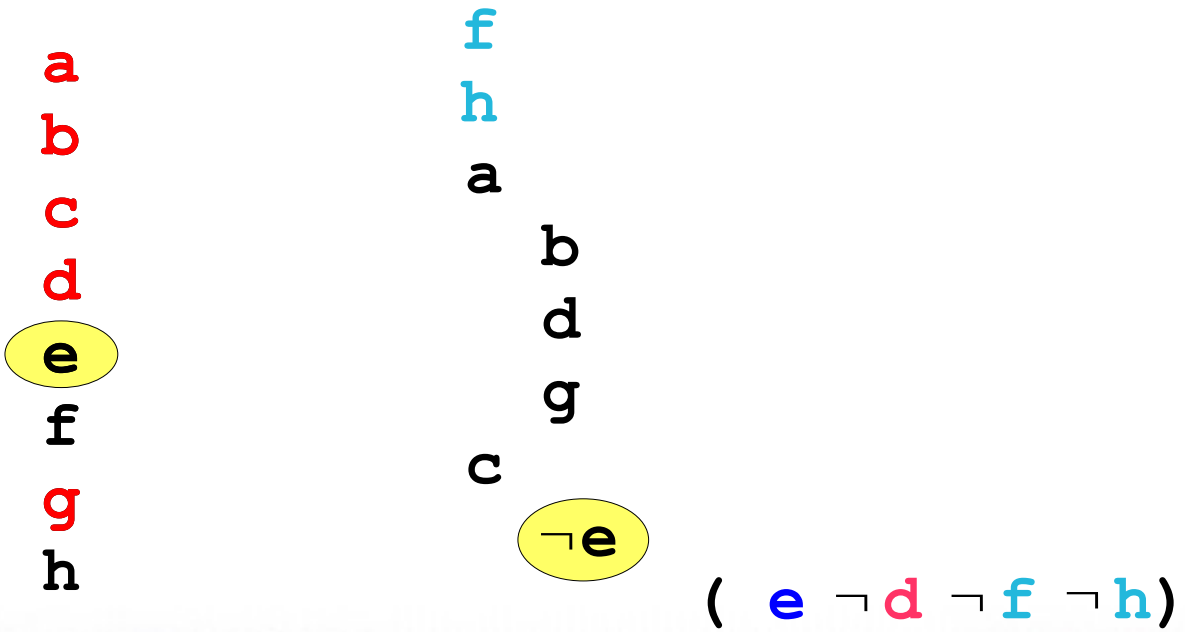

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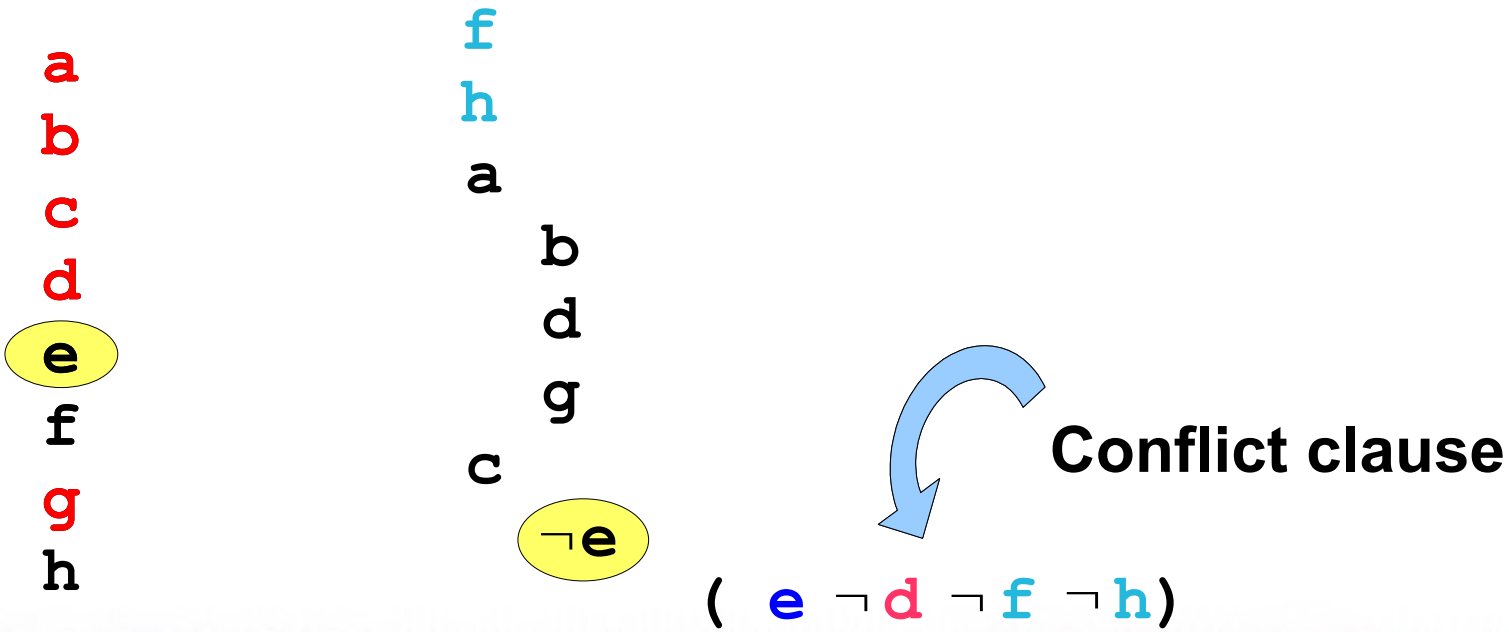

Local Search

- For better clauses, want an implication graph.
- How to generate it from existing assignment I?
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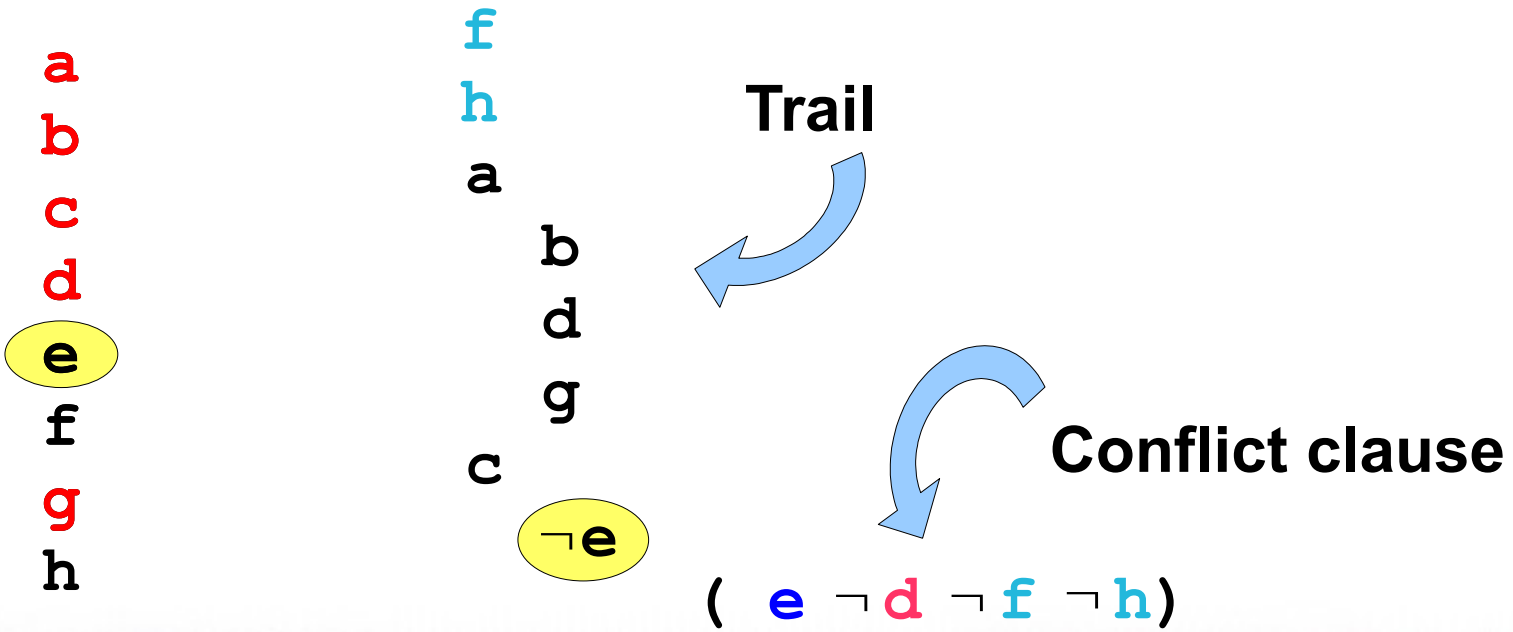

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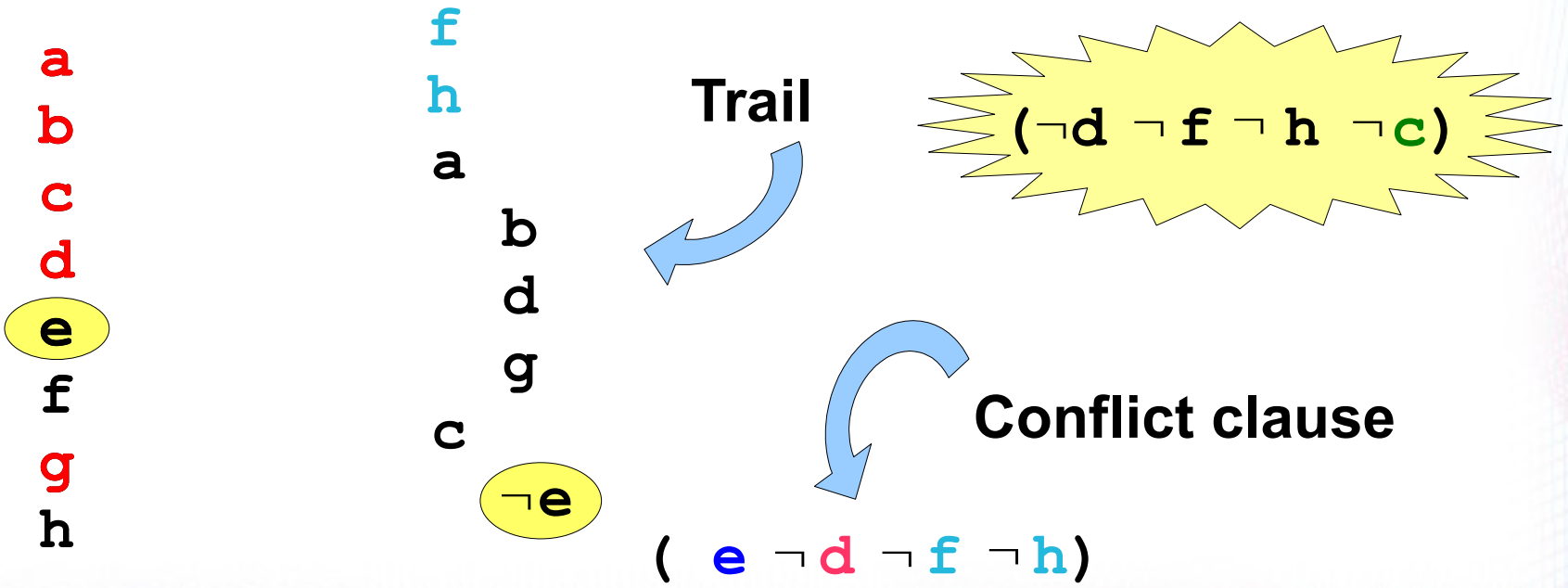

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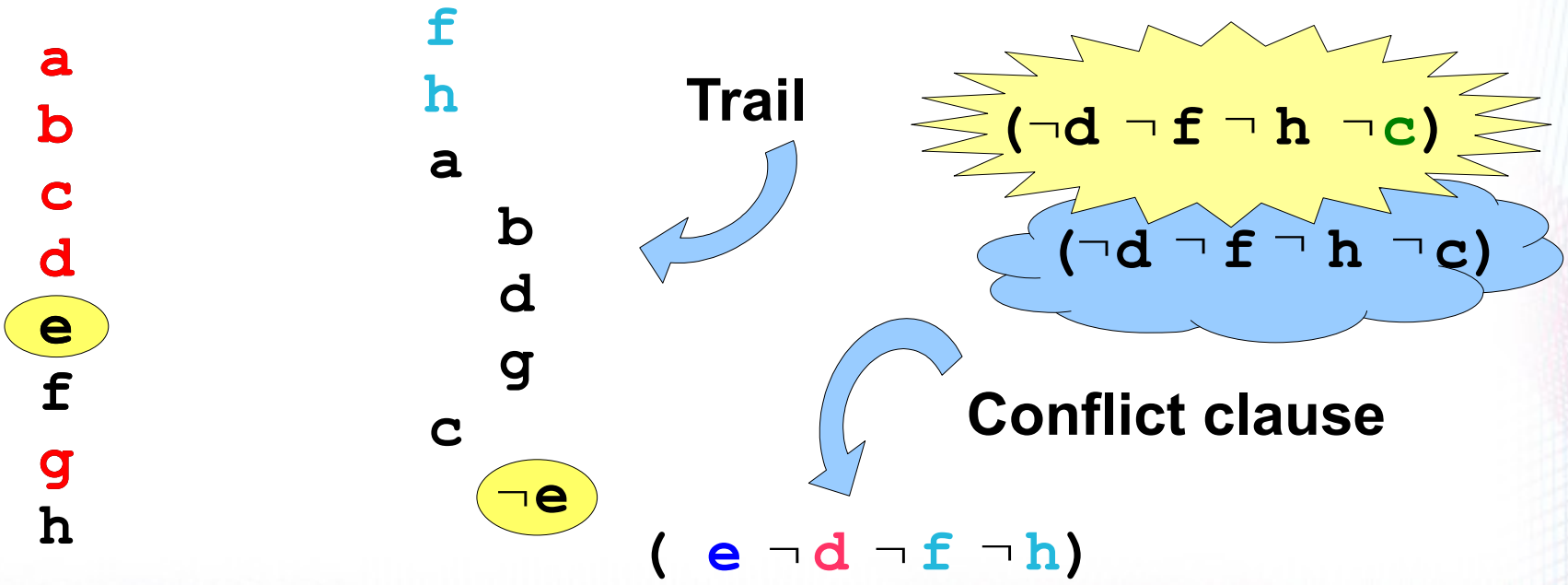
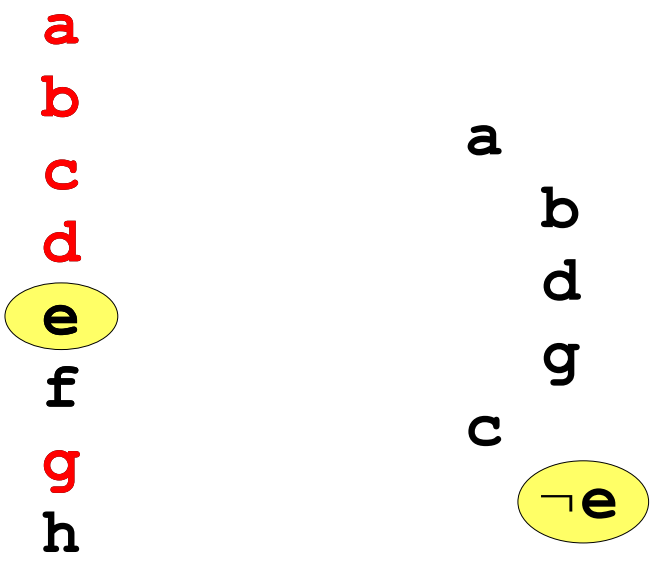


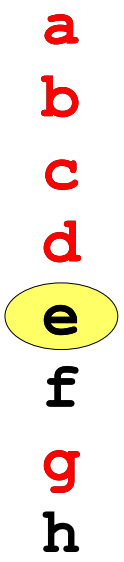
Local Search

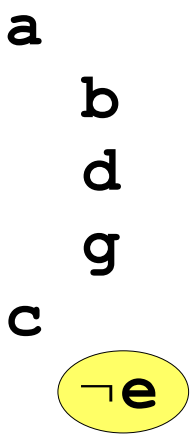
- Problem: this is not a real conflict.
  - A SAT solver would not have learned here.



Local Search

- Problem: this is not a real conflict.
  - A SAT solver would not have learned here.
- You cannot “create” a learnable conflict from an arbitrary assignment





“Unlearnable” assignment

(a, b)
(c, d)
(¬a, c)
(¬b, d)
(¬c, a)
(¬d, b)

¬a ¬b ¬c ¬d

“Unlearnable” assignment

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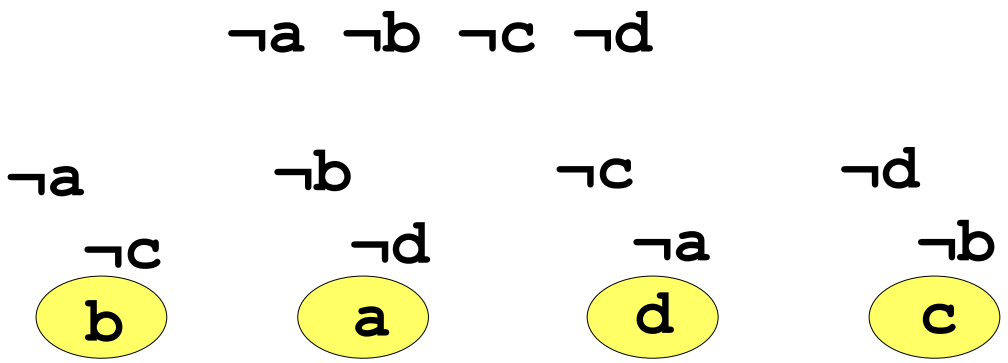

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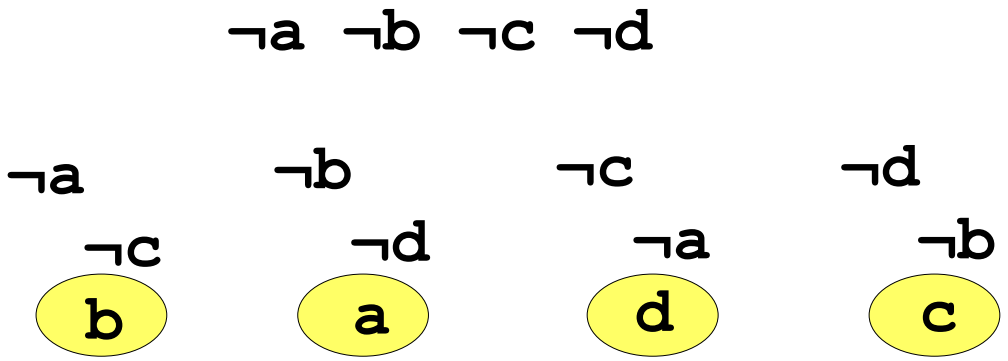

Can we go around this limitation?

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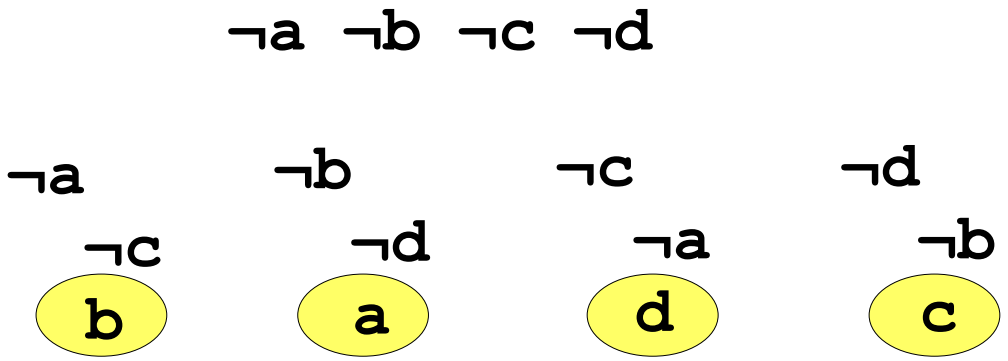

Can we go around this limitation?

- **Allow new values?**

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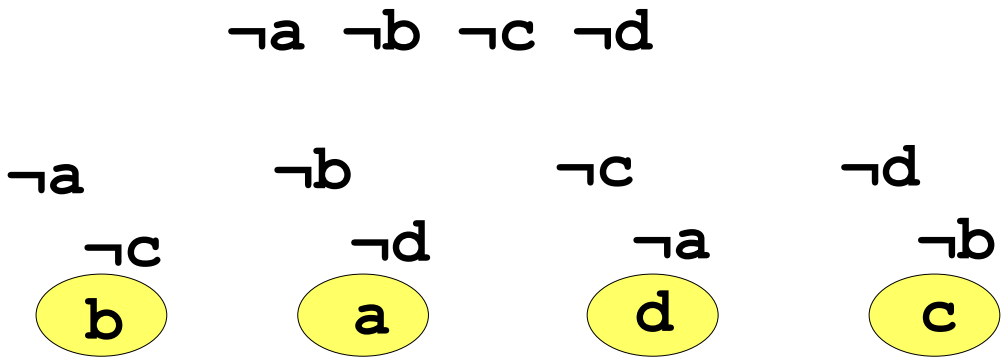

Can we go around this limitation?
- **Allow new values?**
- **Ignore unneeded implications?**

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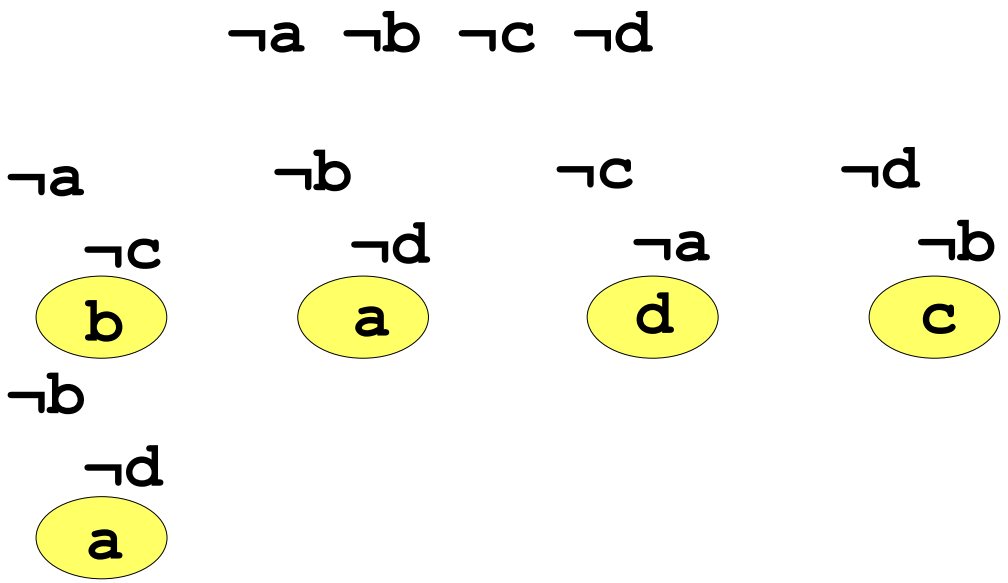

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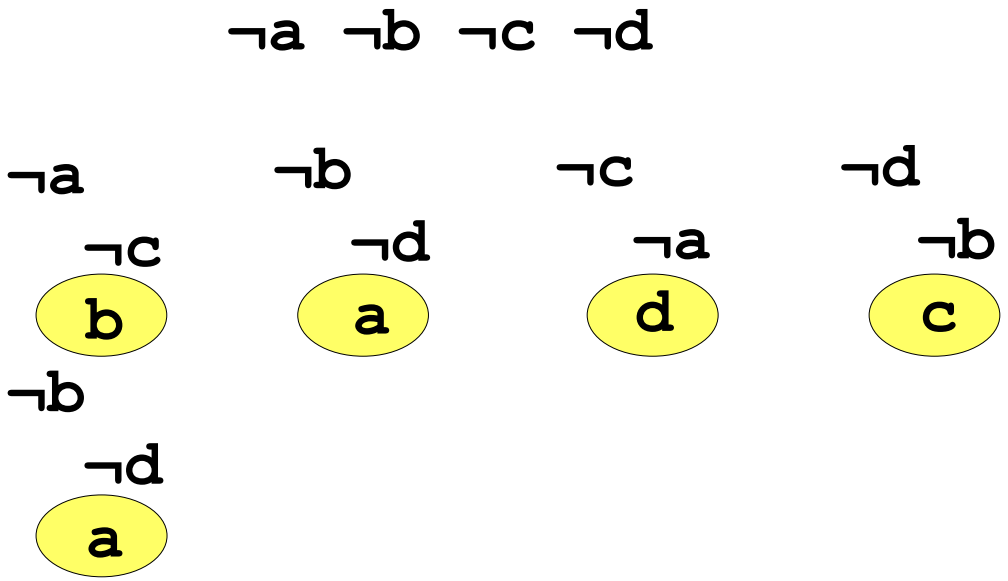

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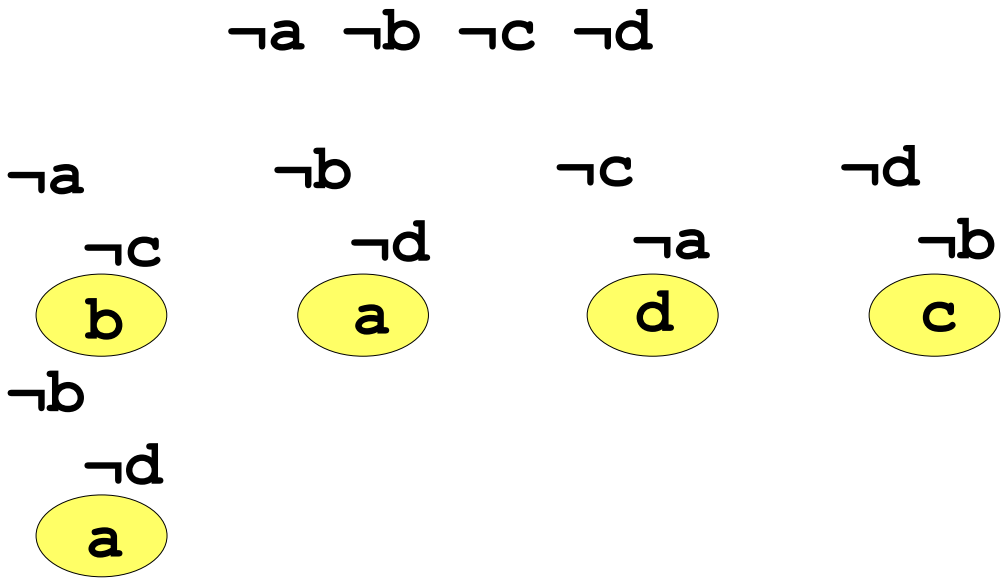


- So: the problem is in the heuristic.
- We don't want “unlearnable” assignments to be a local minima
- *Note*: above assignment is a local minima for clause counting

choosing variables to flip through CDCL

- Inspired by intuition from CDCL solvers
- Uses a base heuristic for variable ordering

  (For eg, VSIDS)
- Guarantees a learnable clause
- Makes the local search solver behave exactly as a CDCL solver

CDCL Local Search

`pickVar(I)`

- Order `I` in a UP-compatible order
  - implied variables have to appear as soon as possible after their reasons
  - (see the paper for more precise definition)
- Pick a false clause C that becomes unit earliest
  - It would have implied some variable v
- If ¬v is implied earlier, have a real conflict.
  - learn a clause from C using the above ordering
- Otherwise, flip v.

Exploiting the insight

- A more efficient way to select variables to flip
  - Vector or vectors instead of a trail?
  - Completely different implementation?
- More freedom
  - Generating other empowering clauses and/or multiple ones from the same assignment?

Learning multiple clauses at once

- Preliminary work
  - currently no way to estimate which are “good” clauses in this context
  - doesn't provide empowerment guarantees
- First conflict clause at every decision level
  - if α does not derives any failed implications, “asserting” clause $(\alpha \rightarrow y)$ is empowering

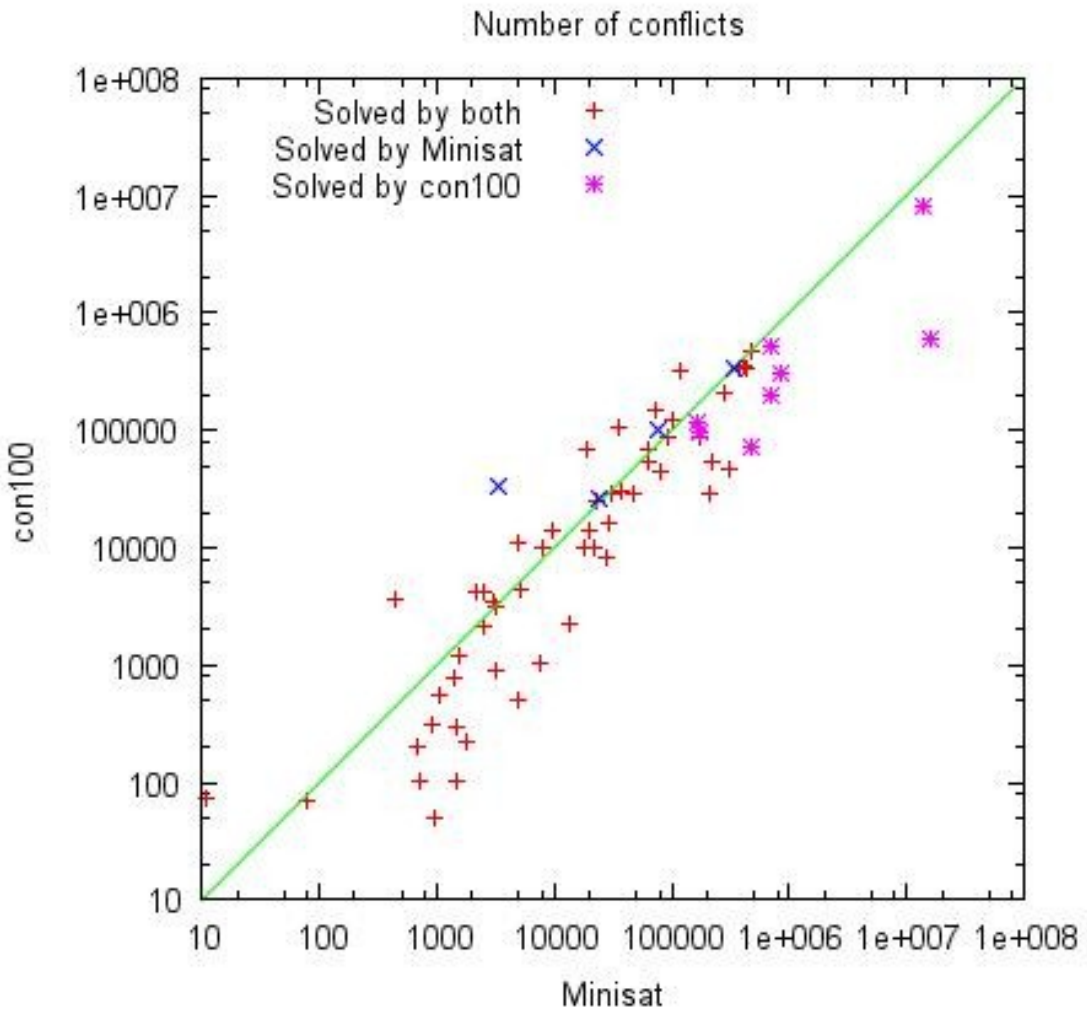
Learning multiple clauses at once

Number of SAT11 problems solved within 1000s: **A**ll, **T**rue, **F**alse instances

Family	Minisat			C_1			C_5			C_10			C_100			C_1000			C_All		
	A	S	U	A	S	U	A	S	U	A	S	U	A	S	U	A	S	U	A	S	U
fuhs (34)	10	**9**	1	10	8	2	9	8	1	9	8	1	10	7	**3**	10	8	2	7	7	0
manthey (9)	3	**3**	0	2	2	0	2	2	0	3	**3**	0	3	**3**	0	1	1	0	0	0	0
jarvisalo (47)	24	8	**16**	24	8	**16**	24	9	15	24	**10**	14	25	9	**16**	17	6	11	15	6	9
leberre (17)	11	4	7	13	**6**	7	14	**6**	**8**	13	**6**	7	13	**6**	7	12	5	7	7	1	6
rintanen (30)	9	**7**	2	8	5	**3**	7	4	**3**	8	5	**3**	8	5	**3**	2	1	1	1	0	1
kullmann (13)	2	2	0	3	3	0	3	3	0	2	2	0	**4**	**4**	0	3	3	0	3	3	0
Total (150)	59	33	26	60	32	28	59	32	27	59	**34**	25	**63**	**34**	**29**	45	24	21	33	17	16

- **C_All learns all clauses all the time**
- **C_n learns all clauses n times every restart, standard learning at other times**

Learning multiple clauses at once



Future work

- A more efficient way to calculate the heuristic
- More freedom
  - Generating other empowering clauses and/or multiple ones from the same assignment?
- Relaxing the restrictions in QBF?
- Other local search heuristics that guarantee “real” conflicts?
- Local search “base” heuristics?

Conclusion

- Reformulated CDCL algorithm as local search
- The trail is simply an efficient way for computing the heuristic
- New view to be explored, and more flexibility to be investigated
  - learning other clauses?
  - better ways of computing the heuristic?
  - flexibility for QBF?