SATLab: X-Raying Random k-SAT (Tool Presentation)

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Standard Random k-SAT

$$\begin{array}{ccccc} x_3 & \vee & \neg x_6 & \vee & x_7 \\ \neg x_{12} & \vee & x_1 & \vee & \neg x_2 \\ & & \ldots & & \\ x_5 & \vee & x_{18} & \vee & x_3 \end{array}$$

- n variables
- $c * n$ clauses of size k
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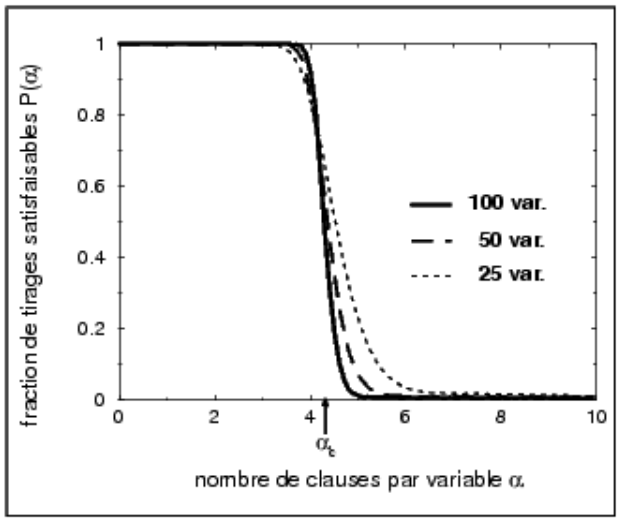
- $c * n$ clauses of size k
- $\Longrightarrow$ kcn literals drawn uniformly and independently
- is there a boolean satisfying assignment?

Why Study Random k-SAT?

- empirically hard on average for $k \geq 3$
- empirical phase transition



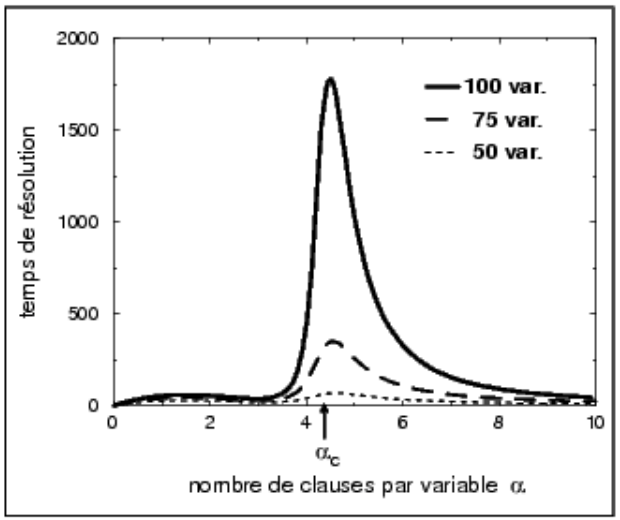


Figure: 3-SAT: satisfiability probability and time complexity. In *Transitions de phase et complexité en informatique*, de G. Biroli, S. Cocco et R. Monasson

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- find the exact location of the threshold:
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Theoretical Challenges

- find the exact location of the threshold:
  - for 3-SAT: between 3.52 and 4.49
- understand the structure of the solutions space:
  - clustering phenomenon?
  - existence of non-trivial cores?
- prove hardness of k-SAT (for $k \geq 3$)

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- average behavior $\neq$ typical behavior
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- maths $\rightarrow$ expectation but not typical behavior
- $\Longrightarrow$ nurtering intuition through experiments

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  - standard, planted, NAE-SAT...

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  - CPU time, binary solutions...
- plot a graph of the results
- $\Longrightarrow$ what SATLab does!

Example: Solvers Competition

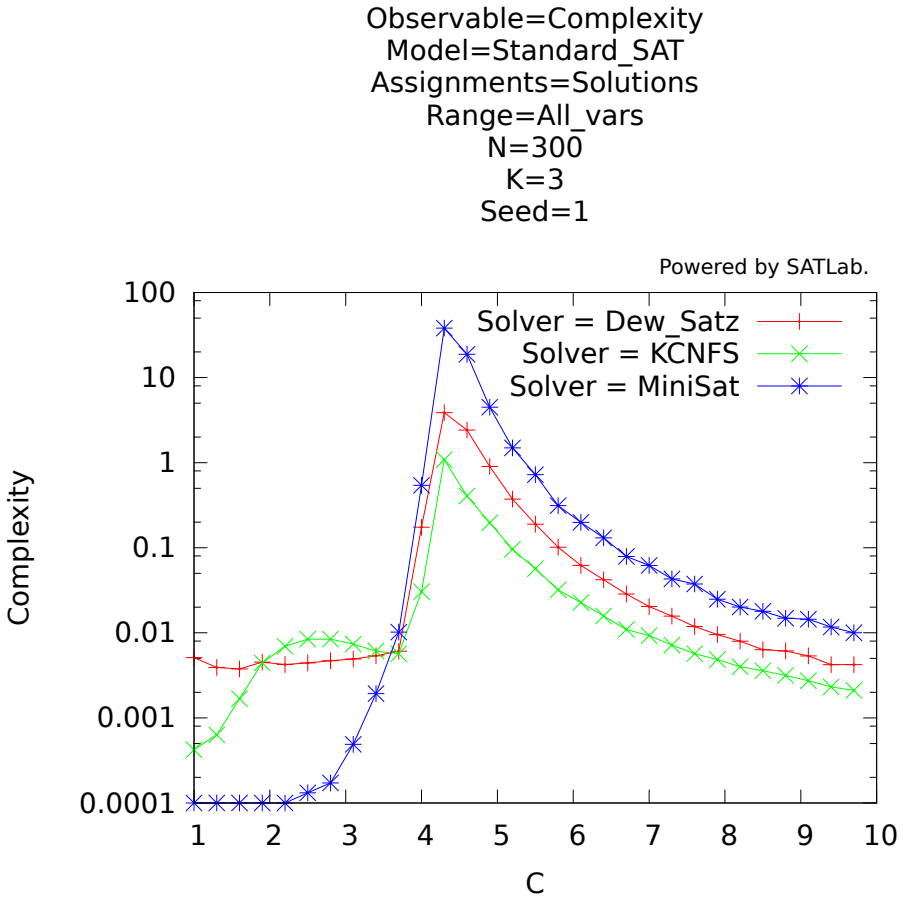

Figure: Peak in time complexity for various solvers.

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- contextual help by right-click

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- SATLab available for download

Example: Clustering

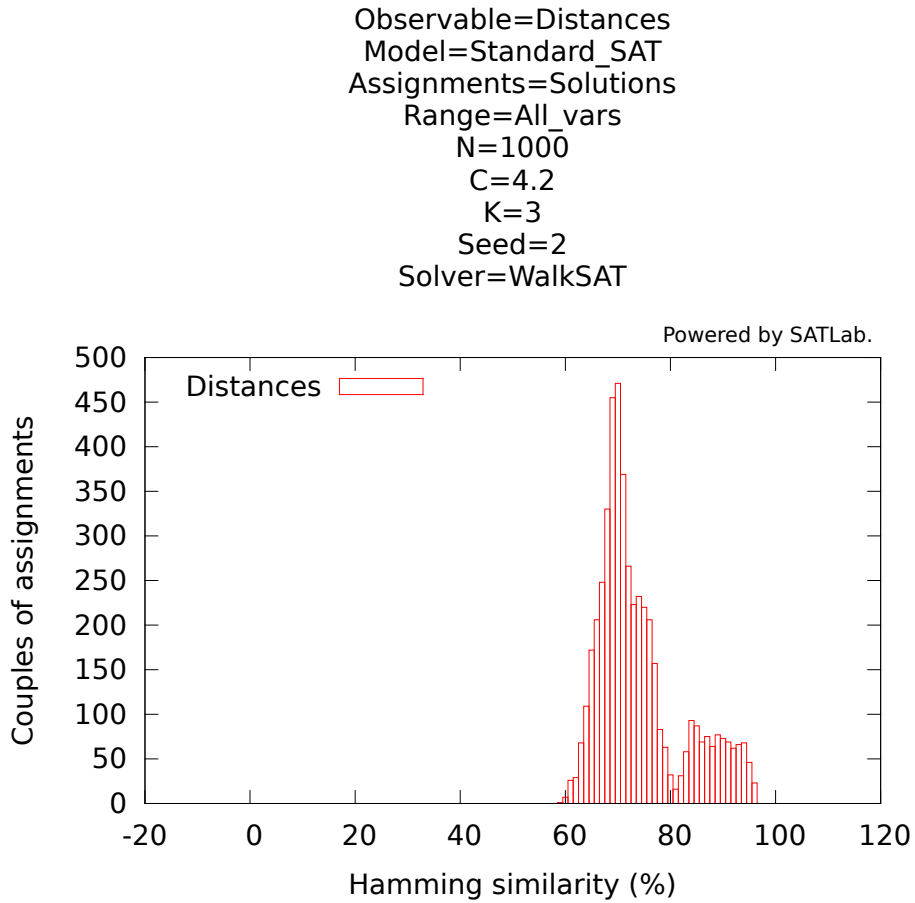

Figure: Hamming similarity between 2 solutions = proportion of variables having the same value in both solutions

Example: Frozen Variables

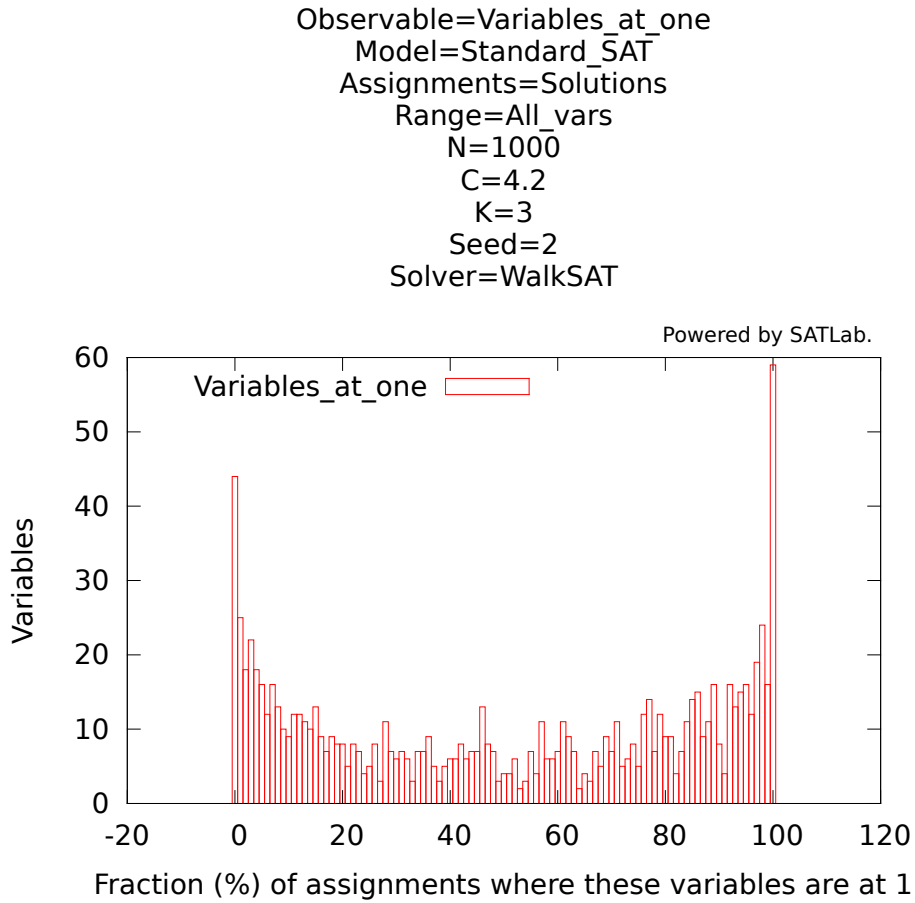

Figure: Number of variables assigned 1 in a given proportion of solutions. Frozen variables appear at 0 and 100.

Example: Cores

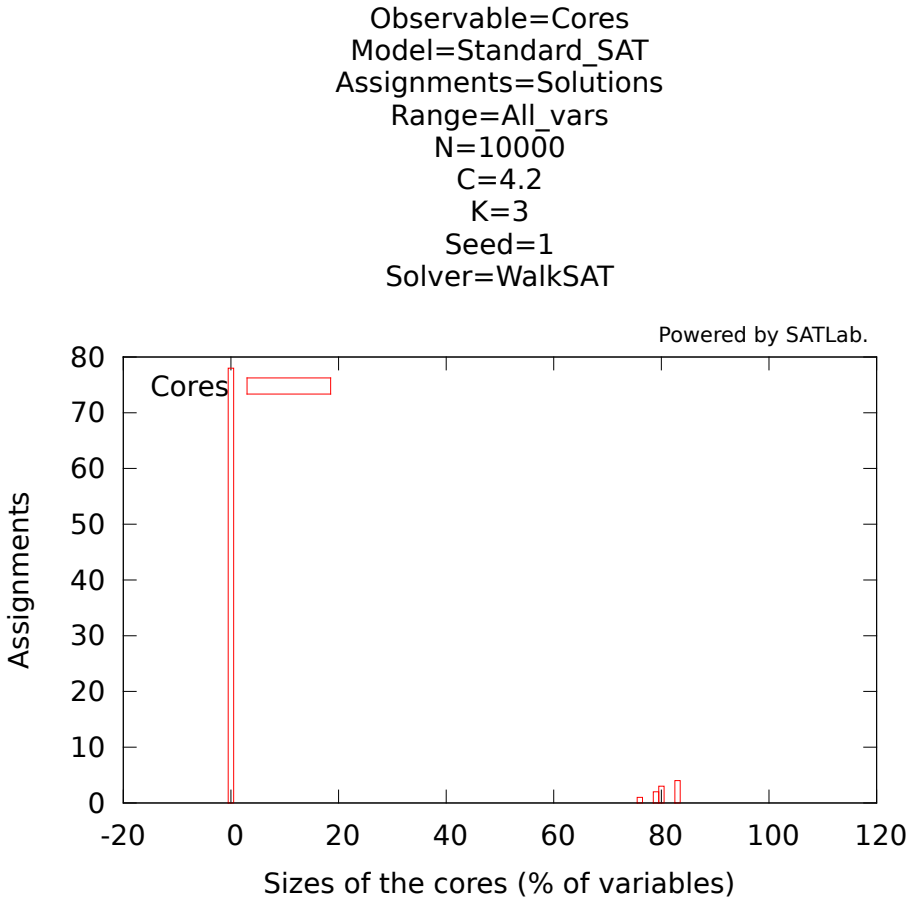

Figure: Size of the cores (% of variables).

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- in 3-SAT, non-trivial cores do not exist when $c \geq 4.42$
- if non-trivial cores are shown to exist
  in the satisfiable area:
- then 4.42 becomes a new upper bound
  on the satisfiability threshold (instead of 4.49)

Acknowledgments

- Yacine Boufkhad, my PhD Advisor
- University Paris Diderot, where I mainly developed SATLab

Perspectives

- add the possibility to analyze non-random instances
- make SATLab a collaborative tool

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- add the possibility to analyze non-random instances
- make SATLab a collaborative tool
- $\Longrightarrow$ your help needed!