

SMT-RAT: An SMT-Compliant Nonlinear Real Arithmetic Toolbox

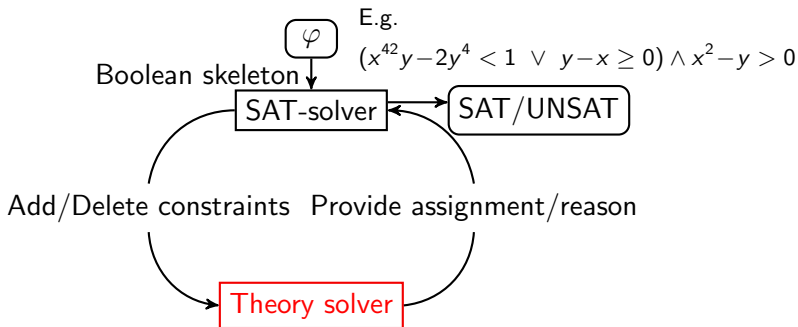
Florian Corzilius, Ulrich Loup, Sebastian Junges, and
Erika Ábrahám



Germany

SAT 2012

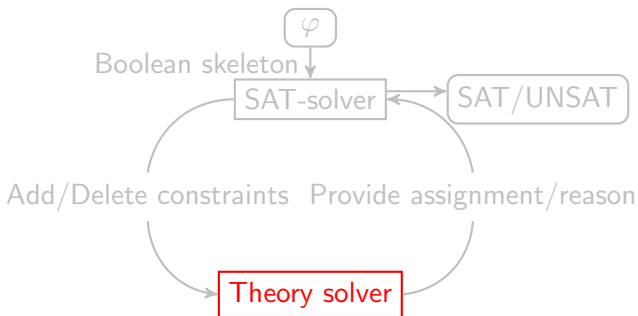
Satisfiability-modulo-theories (SMT) compliant solving



Solves conjunctions of nonlinear real arithmetic (NRA) constraints

- ▶ **incrementally,**
- ▶ **supporting backtracking,**
- ▶ **generating minimal reasons.**

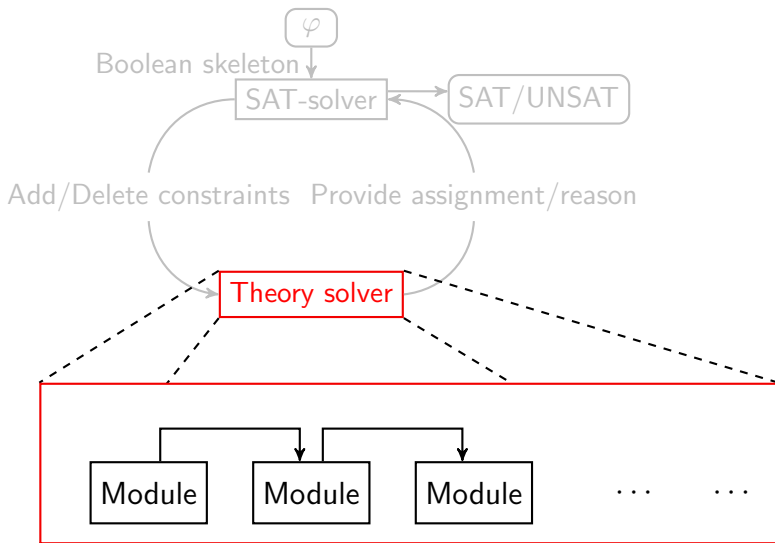
SMT-RAT theory solver



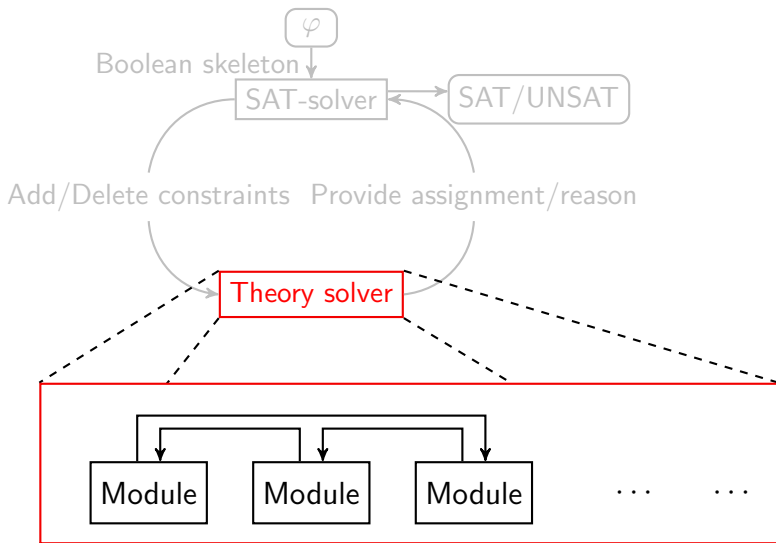
Solves conjunctions of nonlinear real arithmetic (NRA) constraints

- ▶ incrementally,
- ▶ supporting backtracking,
- ▶ generating minimal reasons.

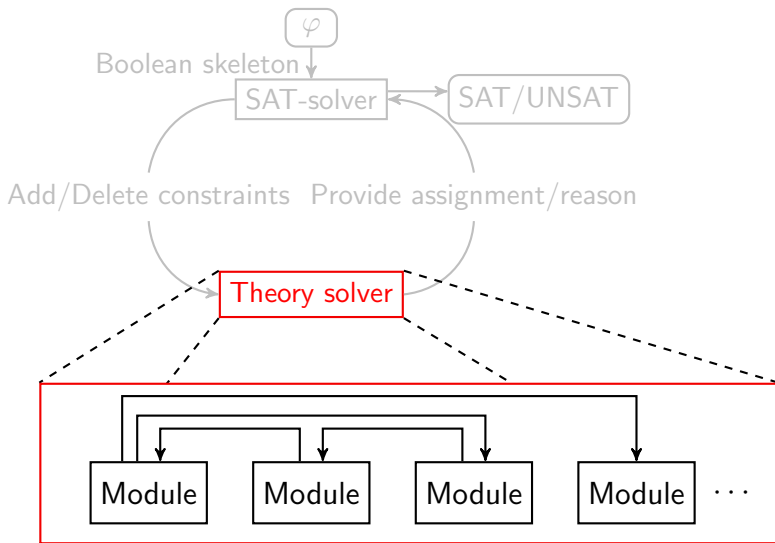
SMT-RAT theory solver



SMT-RAT theory solver



SMT-RAT theory solver



Methods available in SMT-RAT modules

Virtual substitution (VS) [W97]

- ▶ bounded degree of input polynomials, but efficient in practice
- ▶ SMT compliant [ÁCL10, ÁC11]

Methods available in SMT-RAT modules

Virtual substitution (VS) [W97]

- ▶ bounded degree of input polynomials, but efficient in practice
- ▶ SMT compliant [ÁCL10, ÁC11]

Gröbner bases (GB) [DS97]

- ▶ suitable for equations
- ▶ applicable as preprocessor

Methods available in SMT-RAT modules

Virtual substitution (VS) [W97]

- ▶ bounded degree of input polynomials, but efficient in practice
- ▶ SMT compliant [ÁCL10, ÁC11]

Gröbner bases (GB) [DS97]

- ▶ suitable for equations
- ▶ applicable as preprocessor

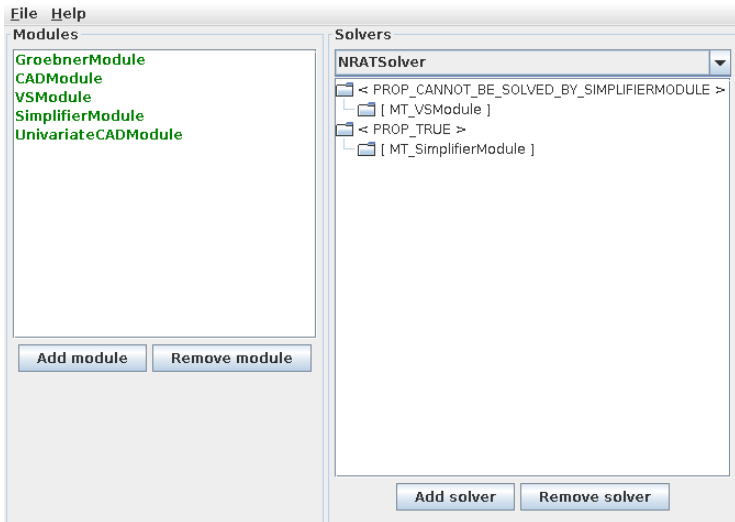
Cylindrical algebraic decomposition (CAD) [C75]

- ▶ complete for NRA
- ▶ often inefficient for equations, but efficient for inequalities

Demonstration

- ▶ Strategy design in SMT-~~X~~RAT
- ▶ SMT-RAT theory solver embedded into OpenSMT

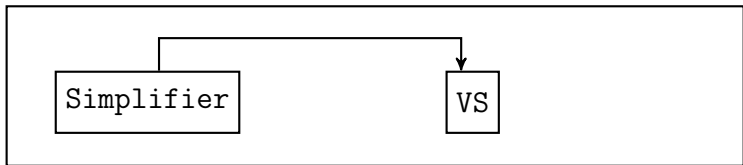
Demonstration



Demonstration 1

Theory call: $\{ x^2 + y^2 + z^2 - 2 = 0,$
 $(x - 1)^2 + y^2 + z^2 - 2 = 0,$
 $(x - 1)^2 + (y - 1)^2 + z^2 - 2 = 0 \}$

Strategy:



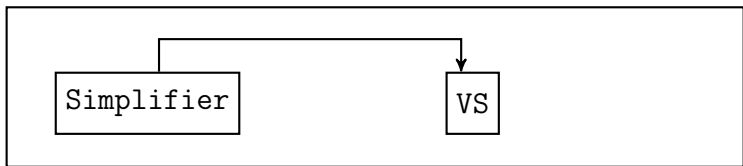
Result: sat

Call trace: Simplifier \rightarrow VS

Demonstration 2

Theory call: $\{ x^2 + y^2 + z^2 - 2 = 0,$
 $(x - 1)^2 + y^2 + z^2 - 2 = 0,$
 $(x - 1)^2 + (y - 1)^2 + z^2 - 2 = 0,$
 $(x - 1)^2 + (y - 1)^2 + (z - 1)^2 - 2 = 0 \}$

Strategy:



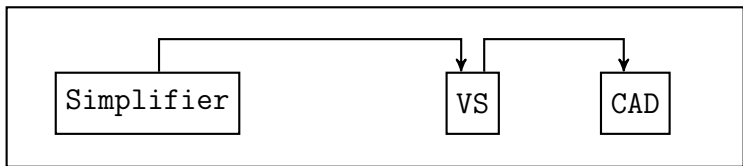
Result: unknown

Call trace: Simplifier \rightarrow VS

Demonstration 3

Theory call: $\{ x^2 + y^2 + z^2 - 2 = 0,$
 $(x - 1)^2 + y^2 + z^2 - 2 = 0,$
 $(x - 1)^2 + (y - 1)^2 + z^2 - 2 = 0,$
 $(x - 1)^2 + (y - 1)^2 + (z - 1)^2 - 2 = 0 \}$

Strategy:



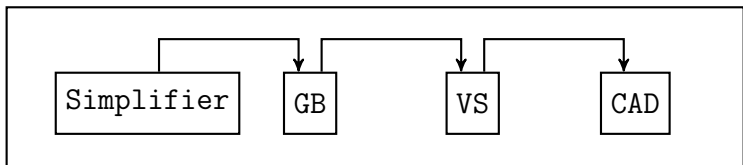
Result: unsat

Call trace: Simplifier \rightarrow VS \rightarrow CAD \rightarrow VS \rightarrow CAD \rightarrow VS \rightarrow CAD

Demonstration 4

Theory call: $\{ x^2 + y^2 + z^2 - 2 = 0,$
 $(x - 1)^2 + y^2 + z^2 - 2 = 0,$
 $(x - 1)^2 + (y - 1)^2 + z^2 - 2 = 0,$
 $(x - 1)^2 + (y - 1)^2 + (z - 1)^2 - 2 = 0 \}$

Strategy:



Result: unsat

Call trace: Simplifier \rightarrow GB

Experimental results

		bouncing ball (52)	etcs (5)	rectanglar pos. (22)	zankl (166)
VS	solved	32	1	14	18
	acc. time	424.02 s	0.35 s	94.07 s	4.42 s
VS +CAD	solved	43	2	16	22
	acc. time	4226.24 s	136.15 s	305.54 s	26.30 s
VS +GB +CAD	solved	43	2	16	22
	acc. time	424.63 s	135.05 s	299.54 s	25.81 s
Z3	solved	0	1	22	62
	acc. time	0.00 s	0.042 s	27.29 s	1138.96 s
CVC3	solved	0	1	0	9
	acc. time	0.00 s	0.11 s	0.00 s	2.86 s






Experimental results

		bouncing ball (52)	etcs (5)	rectanglar pos. (22)	zankl (166)
VS	solved	32	1	14	18
	acc. time	424.02 s	0.35 s	94.07 s	4.42 s
VS +CAD	solved	43	2	16	22
	acc. time	4226.24 s	136.15 s	305.54 s	26.30 s
VS +GB +CAD	solved	43	2	16	22
	acc. time	424.63 s	135.05 s	299.54 s	25.81 s
Z3	solved	0	1	22	62
	acc. time	0.00 s	0.042 s	27.29 s	1138.96 s
CVC3	solved	0	1	0	9
	acc. time	0.00 s	0.11 s	0.00 s	2.86 s



Visit our [SMT-RAT](http://smtrat.sourceforge.net/) on
<http://smtrat.sourceforge.net/>.

References

-  G. E. Collins. *Quantifier Elimination for Real Closed Fields by Cylindrical Algebraic Decomposition*. Springer, Automata Theory and Formal Languages, LNCS 33, 134–183, 1975.
-  A. Dolzmann and T. Sturm. *Simplification of Quantifier-free Formulae over Ordered Fields*. Journal of Symbolic Computation, 1997.
-  V. Weispfenning. *Quantifier Elimination for Real Algebra – The Quadratic Case and Beyond*. Springer, AAECC, 8 (2), 85–101, 1997.
-  U. Loup, F. Corzilius and E. Ábrahám. *A Lazy SMT-Solver for a Non-Linear Subset of Real Algebra*. SMT 2010 (peer-reviewed contribution).
-  F. Corzilius and E. Abraham. *Virtual Substitution for SMT Solving*. Springer, FCT, LNCS 6914, 360 – 371, 2011.