

# Revisiting Clause Exchange in Parallel SAT Solving

Gilles Audemard, Benoît Hoessen, Saïd Jabbour,  
Jean-Marie Lagniez, Cédric Piette

Université Lille-Nord de France  
CRIL - CNRS UMR 8188  
Artois, F-62307 Lens

{*audemard,hoessen,jabbour,piette*}@cril.fr

Institute for Formal Models and Verification  
Johannes Kepler University,  
AT-4040 Linz, Austria

{*jean-marie.lagniez*}@jku.at

June 18, 2012

This work has been supported by CNRS and OSEO, under the ISI project "Pajero".

# SAT framework

# SAT framework

- Lots of solvers are of type *Conflict driven clause learning* (CDCL)

# SAT framework

- Lots of solvers are of type *Conflict driven clause learning* (CDCL)
- CDCL creates one new clause at each conflict

# SAT framework

- Lots of solvers are of type *Conflict driven clause learning* (CDCL)
- CDCL creates one new clause at each conflict
- Restarts are quite frequent

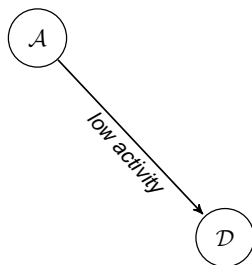
# SAT framework

- Lots of solvers are of type *Conflict driven clause learning* (CDCL)
- CDCL creates one new clause at each conflict
- Restarts are quite frequent
- *Literal block distance (lbd)* provides a qualitative measure about learnt clauses (glucose)

# Freezing learnt clauses

Classical learnt clause  
management

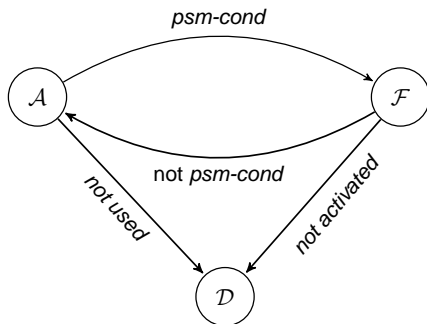
- $\mathcal{A}$ : Set of active clauses  
(used in propagation)
- $\mathcal{D}$ : Set of deleted  
clauses



# Freezing learnt clauses

Freezing learnt clause  
management

- $\mathcal{A}$ : Set of active clauses  
(used in propagation)
- $\mathcal{D}$ : Set of deleted clauses
- $\mathcal{F}$ : Set of frozen clauses





# How can we solve SAT in parallel?

Two main approaches

# How can we solve SAT in parallel?

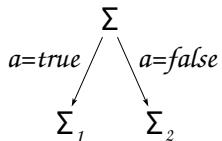
Two main approaches

Divide and conquer

# How can we solve SAT in parallel?

Two main approaches

Divide and conquer

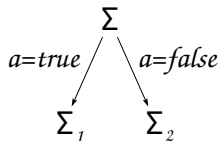


# How can we solve SAT in parallel?

Two main approaches

Divide and conquer

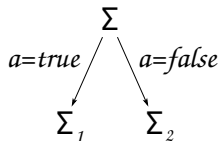
Portofolio



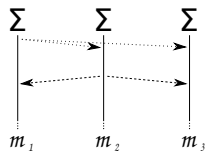
# How can we solve SAT in parallel?

Two main approaches

Divide and conquer



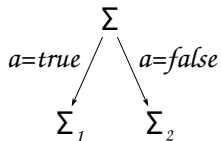
Portofolio



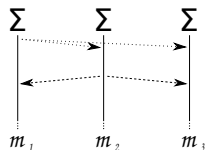
# How can we solve SAT in parallel?

Two main approaches

Divide and conquer



Portofolio



We will use the portofolio methodology

# SAT competition 2011

*ppfolio*

# SAT competition 2011

*ppfolio*

- run completely different state-of-the-art solvers in parallel



# SAT competition 2011

*ppfolio*

- run completely different state-of-the-art solvers in parallel
- trusted the competition (16 medals)

# SAT competition 2011

*ppfolio*

- run completely different state-of-the-art solvers in parallel
- trusted the competition (16 medals)
- the solvers do not communicate!

# SAT competition 2011

## *ppfolio*

- run completely different state-of-the-art solvers in parallel
- trusted the competition (16 medals)
- the solvers do not communicate!

Work needs to be done on communication

# Good communication?

## Good communication?

### Good communication

To achieve good communication, we need to **maximize** the exchange of **useful** information, and **minimize** the **useless** information.

# Communication in portofolio

Good communication

# Communication in portofolio

Good communication

- Information = learnt clauses

# Communication in portofolio

## Good communication

- Information = learnt clauses
- What is a useful clause?



# Communication in portofolio

## Good communication

- Information = learnt clauses
- What is a useful clause?
- A useless clause is never used in propagation

# New ratio

## New ratio

- $\mathcal{I}_t$  the set of imported clauses by thread  $t$

## New ratio

- $\mathcal{I}_t$  the set of imported clauses by thread  $t$
- $used(\mathcal{I}_t, t)$  the number of clauses imported and used in propagation by thread  $t$

## New ratio

- $\mathcal{I}_t$  the set of imported clauses by thread  $t$
- $used(\mathcal{I}_t, t)$  the number of clauses imported and used in propagation by thread  $t$
- $unused(\mathcal{I}_t, t)$  the number of clauses that were deleted by thread  $t$  without being used

## New ratio

- $\mathcal{I}_t$  the set of imported clauses by thread  $t$
- $used(\mathcal{I}_t, t)$  the number of clauses imported and used in propagation by thread  $t$
- $unused(\mathcal{I}_t, t)$  the number of clauses that were deleted by thread  $t$  without being used
- $\#\mathcal{I}_t - used(\mathcal{I}_t, t) - unused(\mathcal{I}_t, t)$  the number of clauses in the database that are neither used, nor deleted by thread  $t$

## New ratio

- $\mathcal{I}_t$  the set of imported clauses by thread  $t$
- $used(\mathcal{I}_t, t)$  the number of clauses imported and used in propagation by thread  $t$
- $unused(\mathcal{I}_t, t)$  the number of clauses that were deleted by thread  $t$  without being used
- $\#\mathcal{I}_t - used(\mathcal{I}_t, t) - unused(\mathcal{I}_t, t)$  the number of clauses in the database that are neither used, nor deleted by thread  $t$

### Usage ratio

$$\frac{\sum_{t=0}^n used(\mathcal{I}_t, t)}{\sum_{t=0}^n \#\mathcal{I}_t}$$

## New ratio

- $\mathcal{I}_t$  the set of imported clauses by thread  $t$
- $used(\mathcal{I}_t, t)$  the number of clauses imported and used in propagation by thread  $t$
- $unused(\mathcal{I}_t, t)$  the number of clauses that were deleted by thread  $t$  without being used
- $\#\mathcal{I}_t - used(\mathcal{I}_t, t) - unused(\mathcal{I}_t, t)$  the number of clauses in the database that are neither used, nor deleted by thread  $t$

### Usage ratio

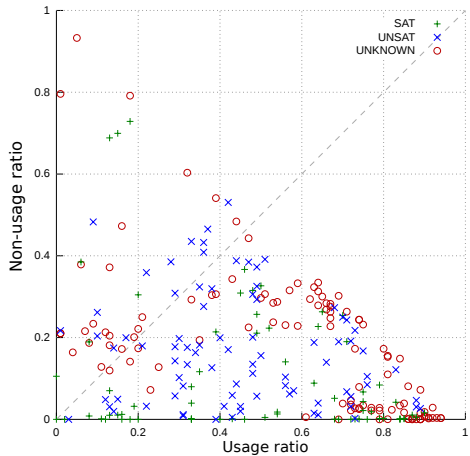
$$\frac{\sum_{t=0}^n used(\mathcal{I}_t, t)}{\sum_{t=0}^n \#\mathcal{I}_t}$$

### Non-usage ratio

$$\frac{\sum_{t=0}^n unused(\mathcal{I}_t, t)}{\sum_{t=0}^n \#\mathcal{I}_t}$$

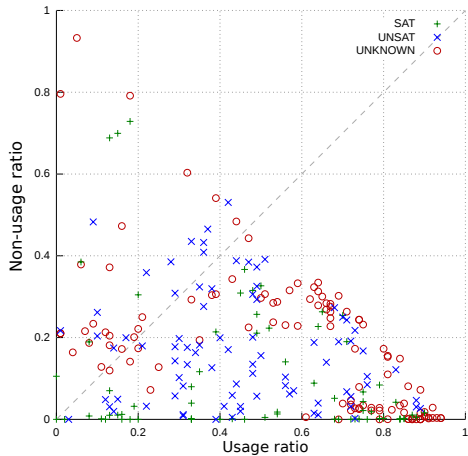


# Classic manySAT



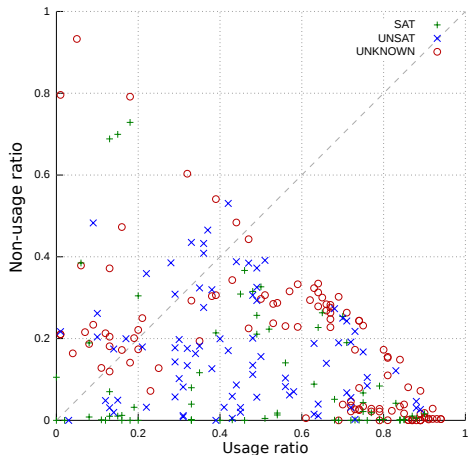
# Classic manySAT

- Ratio is good



# Classic manySAT

- Ratio is good
- A lot of imported clauses are not used but kept in memory



# Challenges

Problems we must face

# Challenges

Problems we must face

- Importation of duplicate information

# Challenges

## Problems we must face

- Importation of duplicate information
- Imported clauses can be useless for the current search subspace

# Challenges

## Problems we must face

- Importation of duplicate information
- Imported clauses can be useless for the current search subspace
- Higher number of learnt clauses

# Introducing PeneLoPe

We want to design a solver based on ManySat 2.0 able to:



# Introducing PeneLoPe

We want to design a solver based on ManySat 2.0 able to:

- handle all the learnt clauses

# Introducing PeneLoPe

We want to design a solver based on ManySat 2.0 able to:

- handle all the learnt clauses
- communicate efficiently

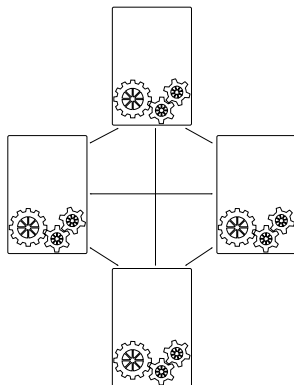
# Introducing PeneLoPe

We want to design a solver based on ManySat 2.0 able to:

- handle all the learnt clauses
- communicate efficiently
- use every processor on the host

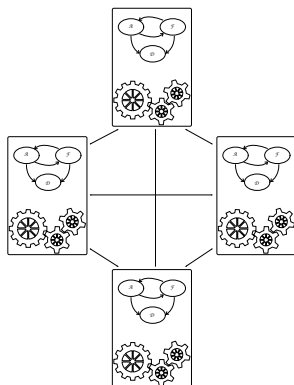
# Freeze in parallel

# Freeze in parallel



# Freeze in parallel

- Each thread has its own sets



# Import policy

# Import policy

- Freeze-all



# Import policy

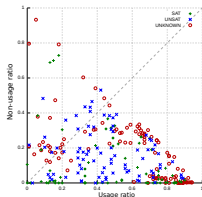
- Freeze-all
- Freeze

# Import policy

- Freeze-all
- Freeze
- No freeze

# Affecting the ratio

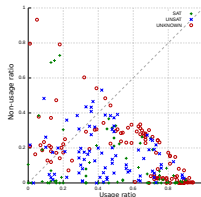
We could change the ratio by:



# Affecting the ratio

We could change the ratio by:

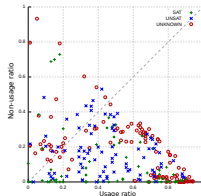
- Restart strategy



# Affecting the ratio

We could change the ratio by:

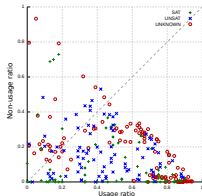
- Restart strategy
  - Luby technique



## Affecting the ratio

We could change the ratio by:

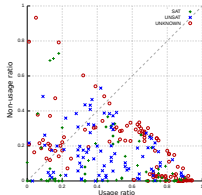
- Restart strategy
  - Luby technique
  - lbd* restarts (*glucose*)



## Affecting the ratio

We could change the ratio by:

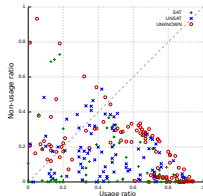
- Restart strategy
  - Luby technique
  - lbd* restarts (*glucose*)
  
- Choosing what is exported



## Affecting the ratio

We could change the ratio by:

- Restart strategy
  - Luby technique
  - lbd* restarts (*glucose*)
- Choosing what is exported
  - Export every generated clauses

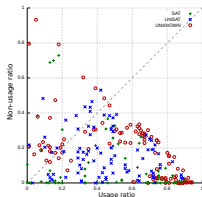




## Affecting the ratio

We could change the ratio by:

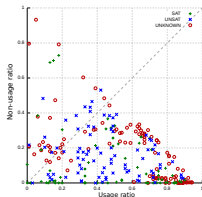
- Restart strategy
  - Luby technique
  - lbd* restarts (*glucose*)
- Choosing what is exported
  - Export every generated clauses
  - Export clauses of size  $\leq s$



## Affecting the ratio

We could change the ratio by:

- Restart strategy
  - Luby technique
  - lbd* restarts (*glucose*)
  
- Choosing what is exported
  - Export every generated clauses
  - Export clauses of size  $\leq s$
  - Export clauses with literal block distance  $\leq l$



## A closer look

SLN: size based export, luby restarts, no freeze at import

LLF: lbd based export, lbd restarts, freeze at import

instance	version	time	$nb_i/nb_c$	$nb_i$	$nb_f$	$nb_u$
<b>hwmcc10-...</b>	SLN	<b>TO</b>	<b>1.34</b>	<b>7989</b>	<b>0%</b>	<b>35%</b>
<b>k50-eijkbs6669-tseitn</b>	LLF	<b>766</b>	<b>4.55</b>	<b>15299</b>	<b>10%</b>	<b>11%</b>
AProVE07-21	SLN	10	1.06	83	0%	35%
	LLF	31	3.53	506	89%	9%

$nb_c$ : number of conflicts (in thousands)

$nb_i$ : number of imported learnt clauses (in thousands)

$nb_f$ : the percentage of learnt clauses frozen at the import

$nb_u$ : the percentage of used learnt clauses

## A closer look

SLN: size based export, luby restarts, no freeze at import

LLF: lbd based export, lbd restarts, freeze at import

instance	version	time	$nb_i/nb_c$	$nb_i$	$nb_f$	$nb_u$
hwmcc10-...	SLN	TO	1.34	7989	0%	35%
k50-eijkbs6669-tseitn	LLF	766	4.55	15299	10%	11%
<b>AProVE07-21</b>	<b>SLN</b>	<b>10</b>	<b>1.06</b>	<b>83</b>	<b>0%</b>	<b>35%</b>
	<b>LLF</b>	<b>31</b>	<b>3.53</b>	<b>506</b>	<b>89%</b>	<b>9%</b>

$nb_c$ : number of conflicts (in thousands)

$nb_i$ : number of imported learnt clauses (in thousands)

$nb_f$ : the percentage of learnt clauses frozen at the import

$nb_u$ : the percentage of used learnt clauses

# Comparison of combinations

## Comparison of combinations

- policies have effects on each other

## Comparison of combinations

- policies have effects on each other
  
- winning policy on our experiments:
  - export: *lbd* based
  - import: *no freeze*
  - restarts: *lbd* based.

## Comparison of combinations

- policies have effects on each other
- winning policy on our experiments:
  - export: *lbd* based
  - import: *no freeze*
  - restarts: *lbd* based.

	#SAT	#UNSAT	#SAT + #UNSAT
Manysat	95	93	188
PeneLoPe			

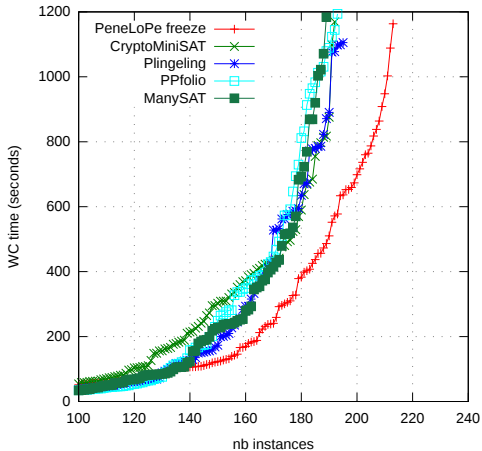


## Comparison of combinations

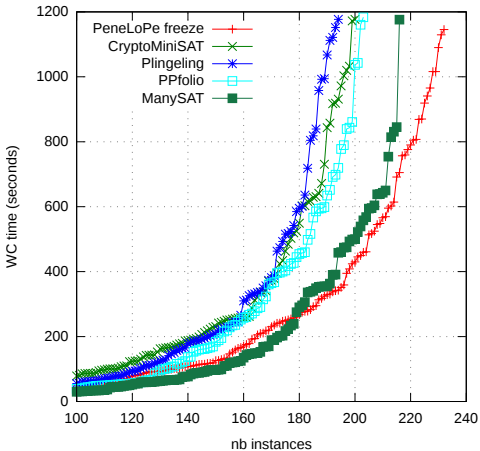
- policies have effects on each other
- winning policy on our experiments:
  - export: *lbd* based
  - import: *no freeze*
  - restarts: *lbd* based.

	#SAT	#UNSAT	#SAT + #UNSAT
Manysat	95	93	188
PeneLoPe	94	111	205

# Comparison with other solvers (8 cores)



# Scaling up to 32 cores



# Conclusion

# Conclusion

- We need to pay attention to clause exchange technique

# Conclusion

- We need to pay attention to clause exchange technique
- The prototype is highly competitive

# Conclusion

- We need to pay attention to clause exchange technique
- The prototype is highly competitive
- We can expend the orthogonality of the threads by using different techniques for each thread

Thank you for your attention



Questions?

# Some comparisons

<i>psm used</i>	<i>export strategy</i>	<i>restart strategy</i>	<i>import strategy</i>	<i>#SAT</i>	<i>#UNSAT</i>	<i>#SAT + #UNSAT</i>
✓	<i>lbd limit</i>	<i>lbd</i>	<i>no freeze</i>	94	111	<b>205</b>
✓	<i>lbd limit</i>	<i>lbd</i>	<i>freeze</i>	89	<b>113</b>	202
✓	<i>size limit</i>	<i>lbd</i>	<i>freeze</i>	93	107	200
✓	<i>size limit</i>	<i>lbd</i>	<i>no freeze</i>	89	107	196
✓	<i>size limit</i>	<i>luby</i>	<i>no freeze</i>	<b>97</b>	98	195
✓	<i>lbd limit</i>	<i>lbd</i>	<i>freeze all</i>	89	102	191
✓	<i>size limit</i>	<i>luby</i>	<i>freeze all</i>	96	92	188
✓	<i>unlimited</i>	<i>lbd</i>	<i>freeze</i>	86	102	188
✓	<i>size limit</i>	<i>luby</i>	<i>freeze</i>	92	96	188
✓	<i>lbd limit</i>	<i>luby</i>	<i>freeze</i>	91	97	188
<i>manysat</i>	-	-	-	95	93	188
✓	<i>lbd limit</i>	<i>luby</i>	<i>no freeze</i>	90	94	184
✓	<i>unlimited</i>	<i>luby</i>	<i>freeze</i>	91	92	183
✓	<i>size limit</i>	<i>luby</i>	<i>no freeze</i>	92	90	182
✓	<i>unlimited</i>	<i>luby</i>	<i>no freeze</i>	89	88	177
✓	<i>size = 1</i>	<i>lbd</i>	<i>freeze</i>	89	88	177

## 8 cores details

Solver	#SAT	#UNSAT	#SAT+#UNSAT
PeneLoPe <i>freeze</i>	97	<b>119</b>	<b>216</b>
PeneLoPe <i>no freeze</i>	96	<b>119</b>	215
plingeling	<b>99</b>	97	196
ppfolio	91	103	194
cryptominisat	89	104	193
ManySat	95	92	187

## 32 cores details

Solver	#SAT	#UNSAT	#SAT+#UNSAT
PeneLoPe <i>freeze</i>	104	127	<b>231</b>
PeneLoPe <i>no freeze</i>	99	<b>131</b>	230
ManySat	105	111	216
ppfolio	<b>107</b>	97	204
cryptominisat	96	105	201
plingeling	100	95	195

# Literal block distance

## Literal block distance

### Definition

Given a clause  $\mathcal{C}$ , and a partition of its literals into  $n$  subsets according to the current assignment, s.t. literals are partitioned w.r.t their decision level. The *lbd* of  $\mathcal{C}$  is exactly  $n$ .

## Literal block distance

### Definition

Given a clause  $\mathcal{C}$ , and a partition of its literals into  $n$  subsets according to the current assignment, s.t. literals are partitioned w.r.t their decision level. The *lbd* of  $\mathcal{C}$  is exactly  $n$ .

### *lbd* restarts

$Avg_s$  is the average of *lbd* of the clauses created since the start of the process.  $Avg_{100}$  is the average of *lbd* over the last 100 created clauses. Restarts when  $Avg_{100} \times \alpha \geq Avg_s$ ,  $\alpha = 0.7$