Fixed-parameter tractability of satisfying beyond the number of variables

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Outline of the Talk

Introduction

- Fixed-parameter tractability
- Some parameterized questions, results related to SAT
- Above guarantee parameterizations
- The main question and the result in the paper
 - Consequences (corollaries)
 - The Algorithm outline
- Conclusions.

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- Input x comes with a parameter k (e.g. solution size, backdoor size to Horn)
- Solve (x, k) in $f(k) + |x|^{O(1)}$ time where f is a function of k alone.
- Parameters can be anything that makes sense in theory and practice (We will see examples shortly).
- Problems that have such an algorithm are said to be fixed parameter tractable (FPT).
- The problem is said to be in XP if it has an $|x|^{f(k)}$ algorithm.
- There is a W-hardness theory to identify problems that are unlikely to have such algorithms

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- *Kernelization*: Reduce (x, k) in polynomial time to an equivalent instance (x', k') such that |x'| and k' are some functions of k.
- Folklore: Fixed-parameter tractable if and only if kernelizable.
- Interesting question: When is |x'| a polynomial function of k?
- Recent breakthroughs in showing lower bounds for kernel sizes (under complexity theoretic assumptions).

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- Given a CNF formula, is there a satisfying assignment with at most k ones (weight at most k)?
 - ▶ W[2]-hard for general SAT (Dominating Set)
 - Fixed-parameter tractable (r^k) for bounded (r) CNF.
 - ▶ For 2CNF, equivalent to VERTEX COVER
- Given a CNF formula, is there a satisfying assignment with weight at least (or equal to) k?
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- Given a CNF formula, is there an assignment satisfying at least *k* clauses?
 - ► (Trivially) Fixed-parameter tractable (with f(k) being c^k where c < 1.4)</p>
- Given a 2CNF formula, is there an assignment that satisfies all but at most *k* clauses? (Hard for 3CNF and above).
 - Fixed-parameter tractable $(15^k - > 9^k - > 4^k - > 2.32^k)$

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Trivially FPT, as if $k \le m/2$, the answer is YES, and otherwise, $m \le 2k$, and we have a kernel for the number of clauses (and one can obtain a kernel for the number of variables with some effort).

- Hence trivial if k is small, and so FPT algorithm applies only when k is large, and the c^k algorithm may not be practical.
- More appropriate question/parameterization: Is there an assignment satisfying at least m/2 + k clauses? FPT (Mahajan and Raman, 1999).

Other 'above guarantee parameterizations' of SAT: Is there an assignment satisfying at least 'Expected number by a random assignment' + k clauses?
 NOT even in XP (under ETH) unless the number of variables in e clause is bounded (or up to log log n)) (LATIN 2023) (=> (=> =

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Let B = (V, F) be the natural variable-clause incidence bipartite graph of the formula F, and let μ be the maximum matching in B.

- Clearly at least μ clauses can be satisfied.
- What about μ + k clauses? It is fixed-parameter tractable to decide whether μ + k clauses can be satisfied (k is the parameter).



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The consequences (Related work)

• For a variable-matched formula (autarky/Crown reduced $\mu = n$), it is FPT to decide whether n + k clauses are satisfiable.

- Minimal unsatisfiable formulae
 - are in general hard to recognize (D^p-complete Papadimitriou and Wolfe);
 - have no crowns, and hence are variable matched (and so one can satisfy at least n clauses)
 - can be recognized in polynomial time if they have n + k clauses (asked by Kleine Buning, shown XP by Kullman, Fleischner and Szeider, FPT by Szeider)
 - This follows as a corollary of our result.

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- Some preprocessing rules
- Branching rules
- Reduce to the m k hitting set problem (Given a family of m sets, hit all of them with a subset of at most m k elements; k is the parameter)

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- Carefully analyzing the drop in (*solnsize matching size*);
- argue that they don't increase in preprocessing steps, and decrease in each branching step.
- Hence need stronger preprocessing rules.
- New (deterministic and improved randomized) algorithms for (m k) hitting set problem

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• P1. If a variable appears only in pure or only in negated form, set it appropriately.

• P2. (Resolution of variables with exactly two occurrences). If a variable x appears once positively and once negatively, then replace the two clauses with their union after deleting x (and \bar{x}).

$$(\mathbf{x} \lor y) \land (\bar{\mathbf{x}} \lor z) \\\downarrow \\ (y \lor z)$$

At this point, every variable appears at least three times.

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Simple preprocessing Rules (Continued

• P3. If there is an autarky (crown), reduce. I.e. if there is a subset S of variables such that $|N(S)| \le |S|$, remove those variables and the clauses containing them.



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Simple preprocessing Rules

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P4. Let there be a subset S of variables such that |N(S)| = |S| + 1. Test whether the clauses of N(S) can be satisfied using variables in S alone.

(Such a subset can be found in polynomial time using matching theory, and sat can be checked in polynomial time using the algorithm for n + 1 clauses satisfiability.)

- If the clauses in N(S) are satisfiable (with variables in S alone), then remove S and its neighbors.
- Else remove variables in S and clauses in N(S) and add a clause that contains all variables in the clauses in N(S) which are not in S.



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At this point, for every subset S of variables $|N(S)| \ge |S|$

(SAT 2012, Trento, Italy)

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Branching Rules

• B1. If x and \bar{x} appear at least twice in F, branch by setting x = 1 or x = 0.

Key argument: While k, the number to be satisfied, drops by at least n(x), the matching size drops by at most n(x) - 1 as the bipartite graph is 2-expanding (i.e. every subset has a buffer of size at least 2).

- Without loss of generality, let n(x) = 1, $n(\bar{x}) > 1$ for every variable x (Otherwise, rename by flipping).
- B2. If a clause contains two positive literals, then branch by setting each of them to 0

As n(x) = 1, this clause is the only clause with positive occurrence for both the variables, so one of them can be false.

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Reduction to m - k hitting set

Where are we?

Every clause has at most one positive literal, and every variable has only one positive occurrence.

• If the clause containing x is (x, C), then replace it by (x) and by adding C to all clauses containing \bar{x} .

$$(x \lor C) \land (\bar{x} \lor \bar{y}) \land (\bar{x} \lor \bar{z}) \land (\bar{x} \lor \bar{w}) \\\downarrow \\ (x) \land (\bar{x} \lor \bar{y} \lor C) \land (\bar{x} \lor \bar{z} \lor C) \land (\bar{x} \lor \bar{w} \lor C)$$

 Resulting formula has a unit clause for each variable, followed by all negated clauses.

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- P1. Set variables appearing in one form (pure or negated)
- P2. Resolve variables appearing once positively and once negatively.
- P3. Reduce autarkies
- P4. Reduce subsets S of variables, where |N(S)| = |S| + 1.
- B1. If a variable appears at least twice positively and negatively, branch.
- B2. If a clause has two positive literals, branch by setting each of them to false.
- Reduce to m k hitting set problem after some transformation.

- (Gutin et al): The family F has a m k hitting set if and only if there is some subset S of the universe such that $|S| \le k$ and S 'hits' at least |S| + k sets.
- Find such a subset (for each size p up to k), if exists, by color coding:
 - Color the sets randomly using p + k colors
 - Find a subset of the universe of size p, if exists, that 'hits' all the colors using dynamic programming
- Derandomize using hash families

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- Showed FPT algorithm for deciding whether a SAT formula has an assignment satisfying at least $\mu + k$ clauses.
- We also show that the problem has no polynomial in k kernel (under complexity theoretic conditions). By a parameter preserving reduction from m - k-hitting set (which has been already shown to have no polynomial kernel under the same conditions)
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Thank You

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